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A Comparison of Two Method for DG Optimizing in Power System

A Thesis

Presented to

The Faculty of the Daniel Felix Ritchie School of Engineering and Computer Science
University of Denver

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by

Yu Li

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Advisor: Wenzhong Gao, Ph.D

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Title: A Comparison of Two Method for DG Optimizing in Power System

Advisor: Wenzhong Gao Ph.D

Degree Date: August 2014

Abstract:

In this paper, we mainly focus on using new method to solve the DG problem in order to reducing the loss. We start from comparing two new method. We list out the mathematics theory of the two methods. And then, we using a 33 bus modeling to show which method has a better function on reducing the loss for system. After that, we take the better method to reducing the loss for two cases (14 bus , 34 bus modeling). At last, we conclude the result of the research and give some comment on the future works.

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Chapter 1: Introduction:

Distributed Generation is never a ignorable problem in the power system production process. We always pay a lot of time and money on the research of this issue. The reason of this is obviously as shown in the following table[1]:

article	millions of people affected	location	Date	Referenc es
Northeast blackout of 1965	30	the United States, Canada	9 Nov 1965	[7]
1999 Southern Brazil blackout	97	Brazil	11 March 1999	[3]
January 2001 India blackout	230	India	02 January 2001	
Northeast blackout of 2003	55	the United States, Canada	14–15 Aug 2003	[5]

article	millions of people affected	location	Date	References
2003 Italy blackout	55	Italy, Switzerland, Austria, Slovenia, Croatia	28 Sep 2003	[6]
2005 Java–Bali blackout	100	Indonesia	18 Aug 2005	[2]
2009 Brazil and Paraguay blackout	817	Brazil, Paraguay	10–11 Nov 2009	[4]
July 2012 India blackout	330	India	30 July 2012	[1]
July 2012 India blackout	670	India	31 July 2012	[1]

Table 1 **list of notable wide-scale power outages**

If we do not or even pay less attention on this issue. The resulted power outage may become a disasters back on us. This paper give us an analytical method on the Distributed Generation and its improvements method. The author hope to put this idea into the analysis of the DG problem in order to make the solution of the DG problem more diversity.

1.1 Writing background:

When the Great scientist Edison invented the light, our life were totally changed. People started to realize the importance of electricity in our life. From then on, people invented a lot of electricity products which have been part of our life, such as air condition, TV and washer. All these makes us relay on the electricity every day. All these bring us to think about how to transmit and distributed and generate the electricity appropriately in order to make us enjoy the electricity more convenient. And what is more, as our construction and production are more and more relay on the electricity in modern time. If we are failed in any part of the electricity production, it will lead to the power outage and a loss of even millions of dollars[2].

From the table1, we could see that the power outage influence more and more people year by year. There is no exactly loss of money. That is because the loss of cost are hard to account. Such kind of data always remind us to pay more attention on the research of the DG. This is the reason we hope to show this new method for solving DG problem.

In this paper, the method is under the theory of improve of analysis(IA) method. The IA method is trying to make calculation process easier by reducing the thesis equations and also to find a more directly analysis to get the result. In this way, we analysis the traditional equations to get the power loss and make a induce in Mathematic to get a new power loss equation. Then, we use a new analysis way of get the optimal DG.

After getting this new method, we use Matlab code and Power World Simulator to prove our idea and get the simulated result. We all know that the Matlab calculation is a perfect tool on the optimization problem. For the Power World, we find this software have a strong function on analysis power flow. We only need to set up the modeling for power system. The result that we could get is clear and direct shown on the bus modeling and also formed in an Extra Excel table.

This paper is with the help of a serious attention and these advanced tools and method.

1.2 Aim of the paper:

In this paper, we mainly focus on how to put a Distributed Generator in the best location of the power system to optimize the efficiency of the system. The Newton-Raphson method is a fast way to get the power flow. In this paper, we use Power world simulator to calculate the power flow. It seems that we avoid the Newton-Raphson method to do the power flow calculation. However, using the Power World simulator to calculate the power flow also need Newton-Raphson method. The simulator put the Newton-Raphson method in its backstage program. When we use the Power World we are using the Newton-Raphson method to do the calculation. Two new method which are similar to each other in math aspect are being bring in to get the optimal location for Distributed Generator.

Why we choose the new method which we called IA(improve of analysis method) instead of the traditional optimal method? In the process of traditional optimal method (no matter the linear optimal method, non linear optimal method or some other search methods), they all have too many constraints which made the calculation much more complicated. The two new methods base on a simple math thesis which we will induce in Chapter 3.

Meanwhile, we use the Newton-Raphson method in getting the base power flow result. We all know that the Newton-Raphson method is the fastest method to calculate the result of power flow. No matter the iteration time or the math function, the Newton-Raphson method always show its obviously advantages for the calculation.

Also, the Distributed Generator in this paper is mainly used on the P-Q bus power system. Beside the slack bus, every bus is P-Q bus in the system which makes the research more focus on the optimization for the power system.

1.3 Structure of the paper:

The Chapter 1 above is the introduction part of the paper. It gives us a background and a reason for write this paper. Following this, we could see the basic knowledge in this paper. They include all the tools, methods and conception of the idea in this paper. In the Chapter 3, we give the details of all the modeling and analysis process in this paper. Then, we list out more description on the tools and

method in the paper. The Chapter 5 is then the results after the simulation with a brief analysis. The Chapter 6 is a conclusion of the paper which includes a small review of the whole paper.

Chapter 2:Basic knowledge

2.1 Distributed generation:

2.1.1 Development of DG:

The Distributed generation is not a new conception for us. The electricity light our life since Edison age. Then, with the highly development of the electricity usage, the Distributed generations become an important issue on the balance, the safety, and the economics of the power system[3].

The development of electricity is too fast that we almost have no time to make an fully arrangement. In the 1870's town gas was transported into most major cities in the U.S and Europe. The following figure is an early power plant(Figure 2-1).



Fig 2-1 Early Power Plant

By the end of 1920's, the utility grids start to become a net that spread widely all over the U.S. From this period of time, the grids construction speed up so fast. which bring the power system up to a higher level. This make people relay much more on electricity. Definitely, it brings up the usage and power load too much for the capacity of power system. In 1934, the public Utility Holding Companies Act of 1934 restrict the companies that could participate the active of the power system construction, distributed , usage. No everyone could now participate into the power system construction , design and production. This leads the power system activities to the large scales age. However, the technology could not support the operation for the large scales of the power system. In 1992, government start to require the interstate

transmission line owners to allow all the electric generators access to their lines. It begin the age that power system needs distributed again. So, when we look back to the history of the distributed generation for system, we could see it is the Distributed-Centralized-Distributed process[2].

2.1.2 Technology of the DG:

The Distributed generation is any small-scale electrical power generation technology that provides electric power at or near the load site; it is either interconnected to the distribution system, directly to the customer's facilities, or both. DG could get us a multitude of services to all the utility and consumers, such as capability adjust, peak sharing, generation allocation confirming, balance of the power system.[4] In addition, the DG could predict the power outage and monitor the status of the power system. As the basic science, more and more new methods are used in the DG technology which bring the DG much more advantage.

In this paper, we mainly focus on how to allocate the DG to balance the power system distribution.

2.2 Improve of analysis (IA):

To solve the problem of the Distributed Generation, many methods have already been used. The major methods are from optimization view. The traditional optimization always needs to set up an objective function. In the DG problem, the objectives are always about cost, power, power loss and so on. Then, we need to find

all the rated parameters for the objective to set up the subjective functions. When we put all them together, we could use Matlab code to realize the calculation. However, when we need to do more than one optimization, such as we need get maximized power Meanwhile the minimized cost. In this case, we will need to set up objective and subjective function for two times. It makes the process of solving the problem more complex. So, later, we bring the GA method in to the DG problem. The GA method is easily for dueling with the multiple objective functions problem. But , The GA method has a big problem. It takes too much time for calculation. These problem and inconvenient make us try to find a better method. So, we start from the optimization itself to see if we could have improve of the method. because the definition of optimization means to find the best values. It is a method to find the extreme .[5] [6]In this way, if we could find the extreme value. We are able to finish the optimal process. think of this ,we could go back the objective function. find the related function for the objective vector. And then , we do some math inducing to get the extreme value. This process is called improve of analysis method.[7]

2.3 Power World simulator:

2.3. 1.Introduction of the Power World:

The Power World Simulator power system simulation visualization software developed by University of Illinois School of Electrical and Computer Engineering can display the visual image of the operation of the power system. Users can create,

change various power system models and parameters for various operations in the emulator. This would analysis the model on failure and economic operation and so on. The design of Power World is user-friendly, and highly interactive nature. It faces to all the user who want to analysis a special power system no matter you are professional or not.

This simulator is a integrated product. Its strong function is on the calculation of the Power Flow. It could calculate a system with even 100,000 nodes. So, it could analysis the system all by itself. Unlike the others software for calculating the Power Flow, the Power World simulator allows users to observe detailed panorama of the power system vividly. In addition, the system model can easily be modified by using the simulation software graphics editing tool. What the user need to do is only click the part that your want to change in the figure of the system. Then, the detail parameter of that part will shown in another window. The user could then revise everything in that window easily. [8]

Power World simulator mostly use the mouse to complete the exchange with the user, the interface follows the practice of using a mouse. In general, the left mouse button can directly modified and control the system equipment while the right can be used to view detailed information about a device, or a list view option. It also provides some special usage on mouse.

Power World simulator many functions can be accomplished by using the toolbar. The toolbar also includes a lot of control bar, which can be activated by the mouse. A total of eight toolbar, as shown below, from left to right, from top to bottom as follows (figure 2-2):[9]



Fig 2-2 Power World simulator functions table

1. File column: Through this column, one can save, print, reading and other operations. This column also provides online help and troubleshooting tool model.

Power World simulator allows users to use several different formats to store the system model. If we look at file in the main menu, then select Save As dialog box items can appear in the lower left corner of the dialog box to store the file type column to select the desired save format. Simulator can be used to store the model in a variety of binary format (the default), the version used in a variety of PTI primitive data types, IEEE common format.

2. Program column: In this column the use could switch the program to edit or run mode in order to control a variety of information in power flow calculation.

This column includes the following options:

a. Edit Mode: Edit mode is used to create new models and modify existing models. Program by clicking on the column "Edit Mode" button (EDIT MODE) could

switch to this mode. In this mode user could do the following operation: Create a new model; increase in existing component models; modify the physical appearance of the one-line diagram; view and modify the model useless chart shows, etc.;

b. Run Mode: This column provides some basic operation button operation mode. You can control the simulation start, continue, pause, and display graphs, switching single line diagrams. In this mode, the timing simulation can be animated, which could achieve single-step flow calculation and the time domain simulation of power systems. In the Options panel click "Run Mode" button to enter the run mode. Key features include: one-line diagram can be viewed through a graphical model; model information (Case Information) shows that by table shows the diagram to see the entire power system; computing trend by changing the simulation options and dialog boxes; translated on any node load, parallel branch and electricity; voltage curves; power transfer distribution factor (PTDF) is calculated; based on trend research conducted purely economic analysis in time domain or user customized, etc.

c. Single step to solve (only in Run mode): single-step flow equations are solved with respect to the timing simulation purposes. This button allows the user to simulate the trend of an independent system.

d. Log: This button can turns on or off the log window. The window displays the progress of the process flow calculation and research to help non-convergence model;

e. Abort: terminate the ongoing flow calculation. If the program is performing a timing simulation, abandoned by (ABORT) key to abort the execution of the program.

3. Image zoom bar: to show the complex power system diagram, Power World simulator can zoom in and move the single-line diagram. One can directly specified in enlarged size in this column ,and could also set the magnification by selecting a rectangular area in Figure. This column contains a dialog box, you can select it as the center of the display to a bus line diagram.

4. Options / Information Bar: Through this column, you can quickly access relevant information and setting options Power World simulator. You can set options related to simulation and solution, define the filter display, single-step flow calculation, create a quick tide tables, display specific information about a node, or switch to another line diagram.

5. Unfolding column: This column provides another move / zoom control. Used to adjust the arrow in Figure 4 of the display range. Another two buttons can be used to scale the image.

6. Format Bar (only in edit mode): Through this column, you can change the font, color, style, graphics and magnification level and so on. The default value may be set different parameters of the drawing, and reset these values when necessary.

Button in this column focused on the main menu, most of the "Format (FORMAT)" item.

7. Edit bar (only in edit mode): This column includes some model editing tools. Can cut or copy a single device on the line diagram and paste it in the original or other figure. Can also be a group of the same elements by the normal operation of the selection rectangle, or circle operation.

8. Insert bar (only in edit mode): Use this column button to add draw on an existing line diagram of the new component. These groups include the element of power system components such as nodes, power lines, transformers, loads, generators and "areas (AREA)", "area (ZONE)" as well as "pie (PIE CHART)" to provide advisory information box pieces. You can also add a text box or display box rectangle, ellipse, arc and other shapes, and these have nothing to do with the system other equipment. Button in this column focused on the main menu, most of the "Insert (INSERT)" item.


2.3.2 Some useful functions:



Double-click Power World Simulator Icon , which starts Simulator.

Simulator can be used to create new models, modify existing models, and electric power system simulation. In the following example, we will create a new model from scratch.[10]

step 1: set up a new folder:

To create a new model from the main menu, select File> New Case , Or click the New icon  on the toolbar. Background window of the screen will turn white, which is modeled Power World default when the background color. As using single line wiring diagram in power system analysis, it uses single line to represent the actual three-phase power system consists of a three-phase component composition.

step2 : add a bus:

- 1.Choose Insert> Bus from the main menu, or bus icon on the toolbar. This is a preparation for adding a new bus for the Simulator

2. Left-click the wiring diagram that you want to place the new bus, it will pop up the Options dialog box (Bus Option Dialog) (as shown in Figure 2-4). This box can be used to define the name of the bus, direction, size, width, area, regional and rated voltage, and is connected to the load and shunt compensator bus.

3. Next, check the bus system balance (System Slack Bus) option, which is located Bus Voltage Bus Information option under item (see Figure 2-5). Bus is used to ensure a balance between supply and demand balance of imaginary power system equipment. In other words, the balance of the bus system balanced the losses caused by the energy difference.

4. Click on the OK button Bus Option Dialog, ending bus definition and close the dialog box. After closing the dialog box, the new bus is displayed in your selected location.

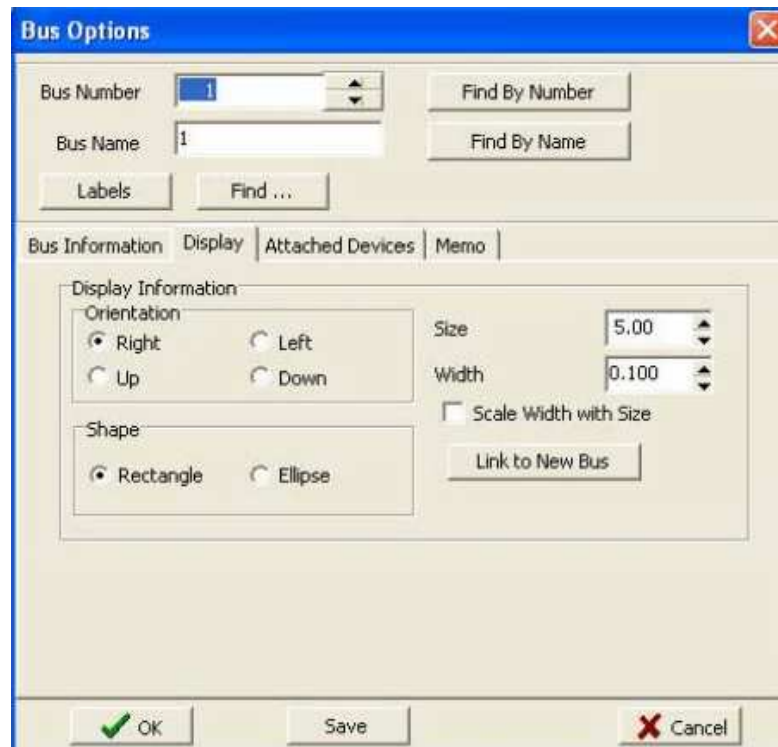


Fig 2-4 Bus Option in power world

Fig 2-5 Important information in Bus Option in power world

step 3: add a generator:

1. Choose Insert> Generator from the main menu Generators icon, or click on

the toolbar 

2. Left-click the bus in wiring diagram need to connect the generator (in this case, click on balancing the bus, that bus One), Generator Options dialog box (Generator Option Dialog) will open automatically (see Figure 2-6). This box set features can be used to determine the new generators, display size, orientation, and its upper and lower output power, reactive upper and lower voltage set point and cost curves.

3 After adding the generator, there should be an output power value defined.

Then, we need to ensure active (MW) and voltage (Voltage) option controls are chose.

Entering '413' in the power output (MW Output) box. Note: The balance of the system bus connected to the generator output power can be Arbitrarily ,as the actual load and the loss of output power of the generator is depending on the system.

4. Clicking display information (Display Information) button. Direction (Orientation) option is used to define the direction of the generator connected to the bus. Lock (Anchored) after a single box is checked, when the bus moves, the generator moves also.

5. Click OK in the generator option (Generation Option Dialog), accept the default values for the remaining options. After the dialog box closed, the new generator will appear in the wiring diagram on the selected bus. The wiring diagram is similar to Figure 2-7

Generator Options

Bus Number: 1 Find By Number Status: ☐ Open ☒ Closed

Bus Name: 1 Find By Name Area Name: 1

ID: 1 Find ... ☒ Same Owner as Terminal Bus

Fuel Type: Unknown Unit Type: UN (Unknown) Labels

Display Information: MW and Voltage Control | Costs | Fault Parameters | Owners, Area, Zone | Memo

MW Control

MW Output: 413 ☒ Available for AGC Part. Factor: 10.00

Min. MW Output: 0.0 ☒ Enforce MW Limits MW Ramp Limit: 20.0

Max. MW Output: 1000.0

Voltage Control

Mvar Output: 0.0 Regulated Bus Number: 1

Min Mvars: -9900 ☒ Available for AVR SetPoint Voltage: 1.0000

Max Mvars: 9900 ☐ Use Capability Curve Remote Reg %: 100.0

MW								
Min Mvar								
Max Mvar								

Fig 2-6 Generator Option in Power World

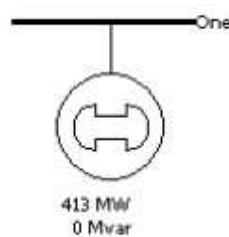


Fig 2-7 Generator figure in Power World

Step 5 Add another stripe load bus

Add the second bus operation

1. Choose From the main menu Insert>Bus, or Click on the icon in the toolbar

for bus bar

2. In the wiring diagram, click on the right side of the first bus somewhere, in figure 2-8. in (bus options dialog), Keep the bus number As the default value 2, and at (bus name) type 'two'

3. At bus bar, add one 200MW, 100Mvar load. Click (attached devices). At (load summery information), type 200 at (base MW), and at (base Mvar), type 100

4. Click ok. Accept all other default values. Closed bus bar options dialog, and Add on the bus

At this moment, on the wiring diagram, don't show the load on bus bar 2. Although in System model file, it has been a load.

On the wiring diagram, the operation to add load is:

1. From the main menu options, choose Insert>Load, or click Load icon

2. Left-click in the middle of the bus. Load options dialog (figure 2-9) is open.

At (constant power), show the 200MW, 100Mvar load

Bus Options

Bus Number: 2 Find By Number

Bus Name: Two Find By Name

Labels Find ...

Bus Information | Display | Attached Devices | Memo

Load Summary Information

Base MW: 200

Base Mvar: 100

Shunt Admittance

G (MW): 0.000

B (Mvar): 0.000

OK Save Cancel

Fig 2-8 bus options dialog in Power World

Load Options

Bus Number: 2 Find By Number

Bus Name: 2 Find By Name

ID: 1 Find ...

Area Number: 1 Area Name: 1

Zone Number: 1 Zone Name: 1

Owner Number: 1 Same Owner as Terminal Bus

Owner Name: 1

Labels

Status: ☐ Open ☒ Closed

Load Information | OPF Load Dispatch

Constant Power Constant Current Constant Impedance

MW Value: 200.000 0.000 0.000

Mvar Value: 100.000 0.000 0.000

Display Information

Display Size: 10.00

☒ Scale Width with Size

Orientation: ☐ Right ☐ Left ☒ Up ☐ Down

Fig 2-9 Load options dialog in Power World

3. Click (load information), choose up to Make the load direction Facing up.

Make sure that (anchored box) is chosen in order to let load move with bus bar.

4. Click ok, accept all other default values. Close dialog and now each Load automatically is equipped with a circuit breaker.

Move components in the wiring diagram

1. Left-click on the element to move, Drag the element to a new location.

2. Move bus bar 2, Left-click on bus bar 2. Drag the element to a new location.

Now the wiring diagram is figure 2-10

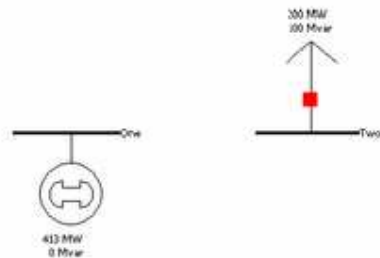


Fig 2-10 Figure for load and generation in Power World

Step 6 add transmission line

Line can connect each bus, and the steps are shown below:

1. In the main menu, choose Insert>Transmission Line, or click (AC transmission line).

2. Left-click on the origin of transmission line, which is usually located in add lines is connected at one end of the bus.

3. Lines and transformer are drawing in series. Click the mouse at a time, Loosen and drag the mouse, a route is extended from the starting point. Click the left mouse button at a time, the line appears a fragment. Every Click the mouse to terminate a line, on a line will appear a vertex. Move the mouse to the next point, between two points will appear a new line in the middle.

4. To end drawing lines, Double-click the left mouse button at the end of hope.

5. (Transmission line/transformer dialog) appears (as in figure 2-11). Number 1 appears in (from bus number) and number 2 appears in (to bus number).

6. (Series resistance), (series reactance) and (shunt charging) are used to Input lines per unit length parameter values. In (resistance), type 0.02, (reactance) 0.08, (shunt charging) 0.1.

7. (Limit (MVA)) is used for power capacity of line. In limit A (MVA), type 1000, other limit (MVA) is for other chapters.

8. Click ok and accept all other default values. Close (transmission line/transformer dialog), new lines are added to the diagram.

Fig 2-11 Transmission line/transformer dialog in Power World

2.4 Optimization:

2.4.1 Description of optimization:

No matter in the basically science such as mathematics, or the application science like computer science and management science. mathematical optimization (alternatively, optimization or mathematical programming) is the selection of a best element (with regard to some criteria) from some set of available alternatives.

In an easy case involved the optimization, the problem always made up by maximizing or minimizing a real function which we could get the input values from within an allowed set and then compute the result for the function.[11] [12]Also, optimization theory and techniques to other formulations include applied mathematics. What is more, optimization needs to reach "best value". Those value for the objective

function need to be constrained in a defined domain. And those include many different kinds of objective function and domains. [13][14][15]

2.4.2 Optimization problem:

The optimization problem is able to be defined as follow: Given: a function $f : A \rightarrow \mathbb{R}$ from some set A to the real numbers Sought: an element x_0 in A such that $f(x_0) \leq f(x)$ for all x in A ("minimization") or such that $f(x_0) \geq f(x)$ for all x in A ("maximization"). Such a formulation is called an optimization problem or a mathematical programming problem (a term not directly related to computer programming, but still in use for example in linear programming – see History below)[16]. Using this technique, the physics and computer problem are mainly focus on energy minimization, making the function f value as energy of the system being modeled.[17][18]

Commonly, A is Euclidean space's subset. This A is always explained by a set of constraints, equalities or inequalities which the A 's members need to match with. The A is named as search space or the choice set.

The function f is called, variously, an objective function, a loss function or cost function (minimization), indirect utility function (minimization), a utility function (maximization), a fitness function (maximization), or, in certain fields, an energy function, or energy functional. A feasible solution that minimizes (or maximizes, if that is the goal) the objective function is called an optimal solution.[19][20][21]

As the convenience consideration, we always translate optimization problem as minimization problem. Commonly, several local minima are existing, expect that both objective function and feasible region are convex in a minimization problem. Here, a local minimum x^* is defined as a point for which there exists some $\delta > 0$ so that for all x such that (2-1):[15][16][22]

$$\|x - x^*\| \leq \delta \quad (2-1)$$

the expression (2-2)

$$f(x^*) \leq f(x) \quad (2-2)$$

hold; it means, all of the function values should be at lease no less to the value at that very point on some region around x^* . It is similar with local maxima.

Many algorithms proposed for solving non-convex problems (including the majority of commercially available solvers) are not good enough for making a difference between local optimal solution and rigorous optimal solutions, and even mistake the former as actual solution to the original problem[23].The branch of applied mathematics and numerical analysis that is concerned with the development of deterministic algorithms that are capable of guaranteeing convergence in finite time to the actual optimal solution of a non-convex problem is called global optimization.[24]

2.5 Power Flow:

2.5.1 Conception:

In power engineering field, the power flow study which is also called load flow study is being used as a tool with numerical analysis in power system. A power flow study prefer simplified notation like one line diagram and per-unit system, and mainly focuses on various forms of AC power (i.e.: voltages, voltage angles, real power and reactive power). It gives an analysis on steady -state operation for power system.

There are a lot of software could solve the power flow problem.[25]

As an addition study of power flow, which is sometimes called the *base case*, some other analysis like short-circuit fault analysis, stability studies (transient & steady-state), unit commitment and economic dispatch are also completed by software. Some programs make use of LP(linear programming) for finding the *optimal power flow*, the conditions that give the lowest cost per kilowatt hour delivered.

Power flow (load flow) studies have important status in further research of power system just like in determining the best operation of existing systems. The principal information that we could get from the power-flow study is the magnitude and phase angle of the voltage at each bus, and the real and reactive power flowing in each line.

Commercial power system are not able solved by hand in power flow calculation only because it is too large. So that, in the year between 1929 and the early 1960s , special purpose network analyzers were set up for building laboratory models of power system which replaced the analog method by large-scale digital computers.

2.5.2 Model:

An *AC power-flow model* is a model used in electrical engineering to analyze power grids. It gives a nonlinear system which describes the energy flow through each transmission line. Due to nonlinearity, in many cases the analysis of large network via AC power-flow model is not feasible, and a linear (but less accurate) DC power-flow model is used instead. Both of those models are very crude approximations to reality.[25]

2.6 Iterative method:

Commonly, we use the iterative method to solve problems of nonlinear programming. And it would be different when the method is used to solve the Hessians, gradients, or only function values. While evaluating Hessians (H) and gradients (G) improves the rate of convergence, for functions for which these quantities exist and vary sufficiently smoothly, such evaluations increase the computational complexity (or computational cost) of each iteration.[26]

One major performance of optimizers is just the number of required function evaluations as this often is already a large computational effort, usually much more effort than within the optimizer itself, which mainly has to operate over the N variables[25]. The derivatives could list out detailed information for such optimizers, and it could be even harder. such as approximating the gradient takes at least $N+1$

function evaluations. The number of function is in the order of N^2 when we get the 2nd derivatives (collected in the Hessian matrix). Newton's method requires the 2nd order derivatives, so for each iteration the number of function calls is in the order of N^2 , but for a simpler pure gradient optimizer it is only N . However, gradient optimizers need usually more iterations than Newton's algorithm. Which one is best with respect to the number of function calls depends on the problem itself.[27]

2.7 Power loss:

The fraction of energy lost to resistance would be reduced when transmitting electricity is at high voltage. However, this varies depending on the specific conductors, the current flowing, and the length of the transmission line. When, think about a known power line, a higher voltage brings the current down and then the resistive losses in conductor. For example, raising the voltage by a factor of 10 reduces the current by a corresponding factor of 10 and therefore the I^2R losses by a factor of 100, provided the same sized conductors are used in both cases. Even if the conductor size (cross-sectional area) is reduced 10-fold to match the lower current the I^2R losses are still reduced 10-fold.[28]

Long-distance transmission always happen through the line overhead with the voltage of 115 to 1,200kV. When the voltage come to more than 2,000kV between the conductor and the ground, corona discharge losses can offset the lower resistive

losses in the line conductors for it is too large. . Measures to reduce corona losses include conductors having larger diameters; often hollow to save weight, or bundles of two or more conductors.

Transmission and distribution losses in the USA is considered to be at 6.6% in 1997 and 6.5% in 2007. We successfully cut in half of the transmission losses by using underground DC. Underground cables, as they have no constraint of light weight than overhead cables. It can be larger diameter. Commonly, losses are estimated from the discrepancy between power produced (as reported by power plants) and power sold to end customers; the difference between what is produced and what is consumed constitute transmission and distribution losses, while we assuming no theft of utility occurs.[29][30]

2.8 Newton- Raphson method:

In calculus, Newton's method is an iterative method for finding zeros (solutions to equations of the form $f(x) = 0$). In optimization, it is applied to the derivative of a function to find its zeros (solutions to $f'(x)=0$), also known as the stationary points of the differentiable function $f(x)$. Newton's Method attempts to construct a sequence x_n from an initial guess x_0 that converges towards x_* such that $f'(x_*) = 0$. This x_* is a stationary point of $f(.)$ [31]

The second order Taylor expansion $f_T(x)$ of a function $f(\cdot)$ around x_n (where

$\Delta x = x - x_n$ is (2-3):

$$f_T(x_n + \Delta x) = f_T(x) = f(x_n) + f'(x)\Delta x + \frac{1}{2}f''(x_n)\Delta x^2, \quad (2-3)$$

and attains its extremum when its derivative with respect to Δx is equal to zero, i.e.

when Δx solves the linear equation (2-4):

$$f'(x_n) + f''(x_n)\Delta x = 0 \quad (2-4)$$

(Considering the right-hand side of the above equation as a quadratic in Δx , with constant coefficients.)

Thus, provided that $f(x)$ is a twice-differentiable function well approximated by its second order Taylor expansion and the initial guess x_0 is chosen close enough to x_* , the sequence (x_n) defined by (2-5):

$$\Delta x = x - x_n = -\frac{f'(x_n)}{f''(x_n)}$$

$$x_{n+1} = x_n - \frac{f'(x)}{f''(x)}, n = 0, 1, \dots \dots \quad (2-5)$$

will converge towards a root of f' , i.e. x_* for which $f'(x_*) = 0$. [32][33][34]

You would probably have some difficulty when you hope to get the root of a complicated function. Lucky, we have Newton-Raphson method. Using this method, we could be easier to get roots of complicated functions in numerically evaluating. .

This iterative process follows a set guideline to approximate one root, considering the function, its derivative, and an initial x-value.

It is known to all that an "0" of the function should be one of a root for the function in mathematics aspect. It is equal to say that the function which equals to "0" is at the location of root. It is no a hard work to find root such as $f(x) = x^2 - 9$ as follow(2-6):[34][35][36][37]

$$f(x) = x^2 - 9 = 0$$

$$(x + 3)(x - 3) = 0$$

$$x = 3 \text{ or } x = -3 \quad (2-6)$$

The Newton-Raphson method start from the iteration. Then, it will gradually come close to a root for function. The root that we get in this way depends on the initial, arbitrarily chosen x-value (2-7).

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (2-7)$$

So, we could get that x_n is the original X-value that we know. $f(x_n)$ is the result value when we give the x_n to function. $f'(x_n)$ is the derivative (slope) at x_n . x_{n+1} here is the next x-value that you are searching for in the iteration process. Here, we should pay attention that as the $f'(x)$ equals to $f(x)/dx$, the $f(x)/f'(x)$ could get a value of dx.

The dx will be close to 0 if the iteration number increases. It is crucial if the Newton-Raphson method is performed. That is what we need to pay attention to. Take the $f(x) = x^2 - 4$ as example. Below are listed the values that we need to know in order to complete the process (2-8).

$$f(x) = x^2 - 4$$

$$f'(x) = 2x \quad x_0 = 6 \quad (2-8)$$

Theoretically, the more uncountable number on iterations we do, the more exactly we could reach to the root for function. Actually, this is not what we hope to use because it is a numerical method that we use to search for the root in a easy way. So, we will assume process has worked accurately when our Δx becomes less than 0.1. Such kind of precision should be different in different situation. So, how much exact we need to use is different depending on each situations. The table (Table 2-1) below list out the execution of the process.

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	dx
0	$x_0 = 6$	$f(x_0) = 32$	$f'(x_0) = 12$	$x_1 = 3.33$	
1	$x_1 = 3.33$	$f(x_1) = 7.09$	$f'(x_1) = 6.66$	$x_2 = 2.27$	$dx = 1.06$
2	$x_2 = 2.27$	$f(x_2) = 1.15$	$f'(x_2) = 4.54$	$x_3 = 2.01$	$dx = .26$

3	$x_3 = 2.01$	$f(x_3) = 0.04$	$f'(x_3) = 4.02$	$x_4 = 2.00$	$dx = 0.01$
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Table 2-1 initial x table

So, starting with $x_0=6$, we could get the root of function $f(x) = x^2-4$ is $x=2$. If we chose another initial x-value, we could find the same root, or we may find the other one, $x=-2$.

A graphical could also help us. In the following graph (Figure 2-12), we could see the function $f(x) = x^2-4$ as blue line. The process we use is as before. IN the first iteration, the red line is tangent to the curve at x_0 . The slope of the tangent is the derivative at the point of tangency, and for the first iteration is equal to 12. Dividing the value of the function at the initial x ($f(6)=32$) by the slope of the tangent (12), we find that the delta-x is equal to 2.67. Subtracting this from six (6) we find that the new x-value is equal to 3.33. Another way of considering this is to find the root of this tangent line. The new x-value (x_{n+1}) will be equal to the root of the tangent to the function at the current x-value (x_n).[38][39][40]

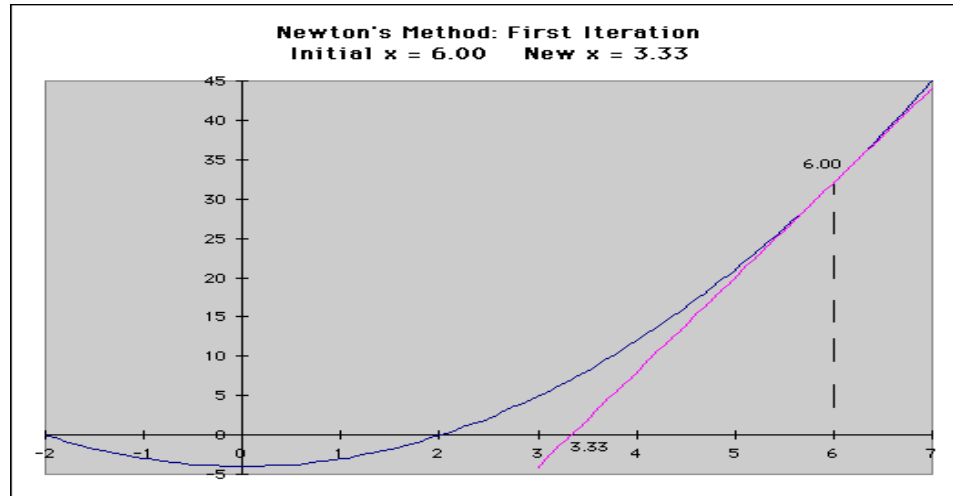


Fig 2-12 Graph for NR method

The Newton-Raphson method is not a "always right" choice. It will bring in some problem while doing the iteration. First, think about the above example. What would be the result if we start from $x=0$? We would have a "division by zero" error, and could not get through. What is more? If you start with $x=1$ to solve the function of $f(x) = x^{1/3}$. Do the x -values converge? Does the Δx decrease toward zero (0)?

So, here, we should pay attention to the using of Newton-Raphson method. When, we focus on some reality project, this problem become more obviously. It is when we are trying to do the derivation we may ignore that the final result may be influenced by many different factors[41].we need to consider all the factor that involved in function for result. They are all the variables. When we think about this situation, we could easily think about using the partial derivatives. So, we

could get a function with all the factor involved as variables and try different partial derivatives to solve the problem to get the result more exactly.

Chapter 3: Modeling:

3.1 DG modeling:

Distributed generation is any small-scale electrical power generation technology that provides electric power at or near the load site; it is either interconnected to the distribution system, directly to the customer's facilities, or both[]. From the previous chapter, we know that many different define were given to the Distributed Generation by many famous agencies. In this paper, The distributed generation are served in for the PQ bus power system as follow:

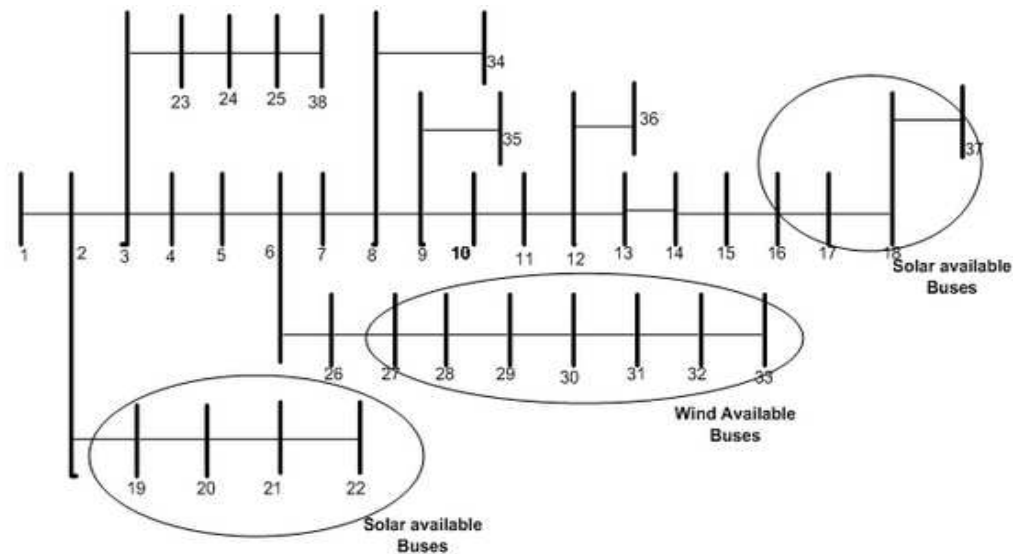


Fig. 3-1 PQ bus mode

In this paper, the Distributed Generation are used to put in a certain bus in the system in order to optimize the efficiency of power system. In the figure above, we could see that the power system has 37bus. Beside the bus1 which is the slack bus, the others are all the PQ bus. The power system we use in paper as the modeling are the same like the figure above. What we have done in this paper is to find the optimal location and size of the Distributed Generator in order to get the optimized power loss later.

3.2 Power loss modeling:

In order to get the math function for the new method which could avoid too many complex constraints. we have two derivation parts. The first one here is to get the power loss function for the new method. The second one is later to getting the optimal size for the system.

3.2.1 Idea of the Power loss equation:

In the original power loss equation, we need to consider the angle, line current and the magnitude of the bus voltage. What we hope to do is do find a equation that related the power loss only to the active and reactive power. So, here, we try to derivate such a loss formula.

3.2.2 Mathematics derivation:

In the [42], the author list out his idea about how to get a new power loss equation. First of all, we start with the network losses by simply adding the bus powers at the n buses of the network. We clear that the bus power S_i injects into bus i which means the generated power minus the bus load. To consider all the bus in the whole system, we could easily get the total generated power minus the total load; i.e., we get the total network losses as follow equation (3-1)(3-2):

$$\begin{aligned} S_1 &= P_1 + jQ_1 \triangleq P_{G1} - P_{D1} + j(Q_{G1} - Q_{D1}) \\ S_2 &= P_2 + jQ_2 \triangleq P_{G2} - P_{D2} + j(Q_{G2} - Q_{D2}) \end{aligned} \quad (3-1)$$

$$P_L + jQ_L = \sum_{i=1}^n S_i = \sum_{i=1}^n V_i I_i^* \quad (3-2)$$

According to the following equation equation(3-3):

$$x^T y \triangleq \sum_{i=1}^n x_i y_i \quad (3-3)$$

The above equation could be just written as following:

$$P_L + jQ_L = v_{bus}^T J_{bus}^* \quad (3-4)$$

By put the follow equations in the above equation:

$$V_{bus} = Z_{bus} J_{bus} \quad (3-5)$$

$$(A + B)^T = A^T + B^T$$

$$(AB)^T = B^T A^T$$

$$(A^T)^T = A$$

$$(3-6)$$

We could get:

$$P_L + jQ_L = J_{bus}^T Z_{bus}^T J_{bus}^* = J_{bus}^T Z_{bus} J_{bus}^* \quad (3-7)$$

The last step follows because Z_{bus} is a symmetric matrix.

In this way, the bus impedance matrix is the sum of bus resistance and bus reactance matrix as follow equation(3-8):

$$Z_{bus} \triangleq R + jX = \begin{bmatrix} r_{11} & \dots & r_{1n} \\ & & \\ r_{n1} & & r_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & \dots & x_{1n} \\ & & \\ x_{n1} & & x_{nn} \end{bmatrix} \quad (3-8)$$

Also, we could list out the bus current vector which is the sum of a real and reactive part vector as following:

$$J_{bus} \triangleq J_p + jJ_q = \begin{bmatrix} J_{p1} \\ \dots \\ J_{pn} \end{bmatrix} + j \begin{bmatrix} J_{q1} \\ \dots \\ J_{qn} \end{bmatrix} \quad (3-9)$$

So, the equation (3-7) could be written as:

$$P_L + jQ_L = (J_p + jJ_q)^T (R + jX) (J_p - jJ_q) \quad (3-10)$$

Focusing on the real part of this matrix product, we could get:

$$P_L = J_p^T R J_p + J_p^T X J_q + J_q^T R J_q - J_q^T X J_p \quad (3-11)$$

As the X is a symmetric matrix, we could easily find that the second and the fourth part of the equation are the same so that we could obtain the next formula for P_L equation(3-12):

$$P_L = J_p^T R J_p + J_q^T R J_q \quad (3-12)$$

or by using **index notation**,

$$P_L = \sum_{k=1}^n r_{jk} (J_{pj} J_{pk} + J_{qj} J_{qk}) \quad (3-13)$$

The above equation shows the total loss power in the manner of the bus current. To us, we are more likely to put the bus power and bus voltage in the equation instead of the current. So, we make a change as equation(3-14):

$$\begin{aligned}
 P_i + jQ_i &= V_i J_i^* \\
 &= V_i (J_{pi} - jJ_{qi}) \\
 &= |V_i| (\cos \delta_i + j \sin \delta_i) (J_{pi} - jJ_{qi}) \quad (3-14)
 \end{aligned}$$

The δ_i is the phase angle of V_i with respect to the reference bus voltage. By separating the real and imaginary parts of the equation(3-14), we could certainly get equation(3-15) as follow:

$$\begin{aligned}
 J_{pi} &= \frac{1}{|V_i|} (P_i \cos \delta_i + Q_i \sin \delta_i) \\
 J_{qi} &= \frac{1}{|V_i|} (P_i \sin \delta_i - Q_i \cos \delta_i) \quad (3-15)
 \end{aligned}$$

If we do a substitution on the equation(3-14) that we put the equation(3-15) above as the current part into this loss equation, we get P_L , after some algebraic step, we get the following express form as equation(3-16):

$$P_L = \sum_{j=1}^n \sum_{k=1}^n \left[\alpha_{jk} (P_j P_k + Q_j Q_k) + \beta_{jk} (Q_j P_k - P_j Q_k) \right] \quad (3-16)$$

For brevity in notation, we introduced two new parameter which will make the equation look more simply:

$$\begin{aligned}
 \alpha_{jk} &\triangleq \frac{r_{jk}}{|V_j||V_k|} \cos(\delta_j - \delta_k) \\
 \beta_{jk} &\triangleq \frac{r_{jk}}{|V_j||V_k|} \sin(\delta_j - \delta_k) \quad (3-17)
 \end{aligned}$$

3.3 Optimization modeling:

Then ,we need to think about how to get the minimized power loss which means how could us optimize the location for the Distributed Generator in order to minimize the power loss.

3.3.1.Idea for the optimization:

We could learn from chapter 2 that we need to find the connect point for the subjective function and objective function. That means we need to push the subjective function on to objective function to make a cross. And as the objective function. It is not too hard to judge that when the cross make only one connect point. It is a special situation. It remains us that when we get the partial derivative of any function and make it equal to zero, we could find the extreme value. Here, if we get this point. It should be the optimal point because the line include the extreme value is tangential to the objective function. So that, this point must be the minimize point that make power loss minimized, as following figure

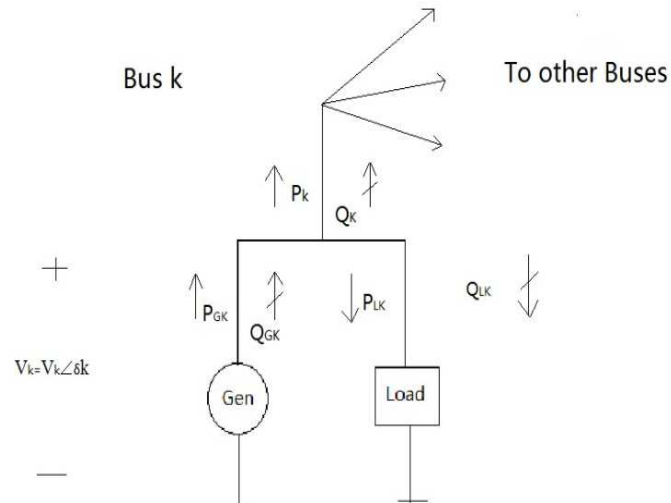


Fig. 3-2 Optimization idea

That is the optimal point for the active power in each bus. And then, as we have the relationship in the bus injection as following figure, we could replace the bus injection (bus) active power with the generator active power.

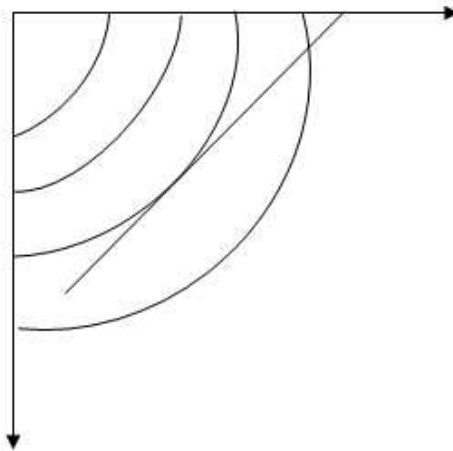


Fig 3-3 Bus node figure

3.3.2 Mathematics derivation:

As we get the idea for derivation the equation for getting the location and size for Distributed Generator, we start with the partial differentiation of power loss with respect to P_{Gi} ; which is eq(3-18):

$$\frac{\partial P_L}{\partial P_i} = \sum_{j=1}^n \frac{\partial}{\partial P_i} [\alpha_{jk}(P_j P_k + Q_j Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k)] \quad (3 - 19)$$

As we know

$$P_i \triangleq P_{Gi} - P_{Di} \quad (3 - 20)$$

The equation(3-19) above is the partial differentiation for real power.

Index		Term			
j	k	$\frac{\partial}{\partial P_i} (\alpha_{jk} P_j P_k)$	$\frac{\partial}{\partial P_i} (\alpha_{jk} Q_j Q_k)$	$\frac{\partial}{\partial P_i} (\beta_{jk} Q_j P_k)$	$\frac{\partial}{\partial P_i} (-\beta_{jk} P_j Q_k)$
j=i	k=i	$2P_i \alpha_{ii}$	0	0	0
j=i	k≠i	$P_k (\alpha_{ik} + P_i \frac{\partial \alpha_{ik}}{\partial P_i})$	$Q_i Q_k \frac{\partial \alpha_{ik}}{\partial P_i}$	$Q_i P_k \frac{\partial \beta_{ik}}{\partial P_i}$	$-Q_k (\beta_{ik} + P_i \frac{\partial \beta_{ik}}{\partial P_i})$
j≠i	k=i	$P_j (\alpha_{ji} + P_i \frac{\partial \alpha_{ji}}{\partial P_i})$	$Q_j Q_i \frac{\partial \alpha_{ji}}{\partial P_i}$	$Q_j (\beta_{ji} + P_i \frac{\partial \beta_{ji}}{\partial P_i})$	$-P_j Q_i \frac{\partial \beta_{ji}}{\partial P_i}$
j≠i	k≠i	$P_j P_k \frac{\partial \alpha_{jk}}{\partial P_i}$	$Q_j Q_k \frac{\partial \alpha_{jk}}{\partial P_i}$	$Q_j P_k \frac{\partial \beta_{jk}}{\partial P_i}$	$-P_j Q_k \frac{\partial \beta_{jk}}{\partial P_i}$

Table 3

However, because the above equation has four parts if we unfold the equation above. we need to think about the relationship between the j, k. The results turn into

many different situations. which is based mainly on the value of j , k that they get. In the above table, we make the summarize.

when we get the table above, we could put them together as the whole partial derivation equation as follow (3-21).

$$\begin{aligned}
\frac{\partial P_L}{\partial P_i} = & 2P_i \alpha_{ii} \\
& + \sum_{\substack{k=1 \\ k \neq i}}^n \left[\alpha_{ik} P_k - \beta_{ik} Q_k + (P_i P_k + Q_i Q_k) \frac{\partial \alpha_{ik}}{\partial P_i} + (Q_i P_k - P_i Q_k) \frac{\partial}{\partial P_i} \right] \\
& + \sum_{\substack{j=1 \\ j \neq i}}^n \left[\alpha_{ji} P_j - \beta_{ji} Q_j + (P_i P_j + Q_i Q_j) \frac{\partial \alpha_{ji}}{\partial P_i} + (Q_j P_i - P_j Q_i) \frac{\partial \beta_{ji}}{\partial P_i} \right] \\
& + \sum_{\substack{j=1 \\ j \neq i}}^n \sum_{\substack{k=1 \\ k \neq i}}^n \left[(P_j P_k + Q_j Q_k) \frac{\partial \alpha_{jk}}{\partial P_i} + (Q_j P_k - P_j Q_k) \frac{\partial \beta_{jk}}{\partial P_i} \right]
\end{aligned} \tag{3-21}$$

If we could understand that

$$\begin{aligned}
\alpha_{kj} &= \alpha_{jk} \\
\beta_{jk} &= \beta_{kj}
\end{aligned} \tag{3-22}$$

We were also able introduce the shorter symbols:

$$\begin{aligned}
\frac{\partial \alpha_{jk}}{\partial P_i} &\triangleq \alpha'_{jk} \\
\frac{\partial \beta_{jk}}{\partial P_i} &\triangleq \beta'_{jk}
\end{aligned} \tag{3-23}$$

The we could reduce the equation(3-21), as

$$\begin{aligned}
\frac{\partial P_L}{\partial P_i} &= 2 \sum_{k=1}^n (P_k \alpha_{ik} - Q_k \beta_{ik}) \\
&= \sum_{j=1}^n \left[(P_j P_k + Q_j Q_k) \right] \alpha'_{jk} - (P_j Q_k - Q_j P_k) \beta'_{jk}
\end{aligned} \tag{3-24}$$

From this equation(3-24) we may find that it is not convenience to do the calculation. We 'd better try to get a clearer expressions for the partial derivatives.

This may not be a difficult thing for us to deal with if we take a look at what we have already done.

With the differentiating the expression for α_{jk} and β_{ik} which is list out by equation (3-22),we get:

$$\begin{aligned}\frac{\partial \alpha_{jk}}{\partial P_i} &\triangleq \alpha'_{jk} = -\frac{r_{jk}}{|V_j||V_k|} \sin(\delta_j - \delta_k) \left(\frac{\partial \delta_j}{\partial P_i} - \frac{\partial \delta_k}{\partial P_i} \right) \\ \frac{\partial \beta_{jk}}{\partial P_i} &\triangleq \beta'_{jk} = \frac{r_{jk}}{|V_j||V_k|} \cos(\delta_j - \delta_k) \left(\frac{\partial \delta_j}{\partial P_i} - \frac{\partial \delta_k}{\partial P_i} \right)\end{aligned}\quad (3-25)$$

we then take the expression for the P_i for the equation that we all know as the following equation(3-26)

$$\frac{P_i - jQ_i}{V_i^*} = y_{i1}V_1 + y_{i2}V_2 + \dots + y_{in}V_n \quad \text{for } i = 1, 2, \dots, n \quad (3-26)$$

we then ,get:

$$P_i = \text{Re}\{\sum_{v=1}^n y_{iv}V_vV_i^*\} \quad (3-27)$$

since:

$$V_i^* = |V_i|e^{-j\delta_i} \quad (3-28)$$

and then

$$V_v = |V_v|e^{j\delta_v} \quad (3-29)$$

and then, when we think about that the elements of the Ybus matrix which could be put in polar form:

$$y_{iv} = |y_{iv}|e^{j\varphi_{iv}} \quad (3-30)$$

then we could transforms equation(3-27) into following (3-31);

$$P_i = \text{Re} \left\{ \sum_{v=1}^n |y_{iv} V_v V_i| e^{j(\delta_v - \delta_i + \varphi_{iv})} \right\} \quad (3-31)$$

By unfolding the real part of the Equation(3-31), we could get:

$$P_i = \sum_{v=1}^n |y_{iv} V_v V_i| \cos(\delta_v - \delta_i + \varphi_{iv}) \quad (3-32)$$

From equation(3-32) we have, upon differentiation:

$$\frac{\partial P_i}{\partial \delta_j} = -|y_{ij} V_j V_i| \sin(\delta_j - \delta_i + \varphi_{ij}) \quad (3-33)$$

we put the above partial derivative part into equation(3-25) and we could

get following final formulas for computation of α'_{jk} and β'_{jk} :

$$\begin{aligned} \alpha'_{jk} &= \frac{r_{jk} \sin(\delta_j - \delta_k)}{|V_i| |V_j| |V_k|} \left[\frac{1}{|y_{ij}| |V_j| \sin(\delta_j - \delta_i + \varphi_{ij})} \right. \\ &\quad \left. - \frac{1}{|y_{ik}| |V_k| \sin(\delta_k - \delta_i + \varphi_{ik})} \right] \\ \beta'_{jk} &= \frac{r_{jk} \cos(\delta_j - \delta_k)}{|V_i| |V_j| |V_k|} \left[\frac{1}{|y_{ik}| |V_k| \sin(\delta_k - \delta_i + \varphi_{ik})} \right. \\ &\quad \left. - \frac{1}{|y_{ij}| |V_j| \sin(\delta_j - \delta_i + \varphi_{ij})} \right] \end{aligned} \quad (3-34)$$

When ,we get the equation above, we could also put the equation() with the above equation together, we could then compute the partial derivate result. However, because the two equations combined together are too long for calculation. This situation need much more computing time. So, we get[]:for the typical system parameter the double sum contributes only an insignificant part of the equation above,

and for the most situation we should therefore be able to use following approximate but timesaving formula for the equation(3-35):

$$\frac{\partial P_L}{\partial P_i} \approx 2 \sum_{k=1}^n (P_k \alpha_{ik} - Q_k \beta_{ik}) \quad (3-35)$$

3.4 Two calculation methods analysis:

From the above derivate process we could get the new optimal way to find the optimized location and size for Distributed Generator. As we induced in the chapter2 , This is the IA(improve of the analysis method) . Now, how to final get the size and location and then to bring them back to the power loss calculation to get the optimized power loss is the problem that we are now trying to figure out. Here we get two analysis methods to get the results.

3.4.1 Method 1 start from analysis the real injection power:

The first method [5]is following the equation(3-35) in 3.3, as we get:

$$\frac{\partial P_L}{\partial P_i} \approx 2 \sum_{k=1}^n (P_k \alpha_{ik} - Q_k \beta_{ik})$$

We could see that there are two different situations in the above equation, that is when $i=j$, and when $i \neq j$. Based on this, we get:

$$\alpha_{ii} P_i - \beta_{ii} Q_i + \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij} P_j - \beta_{ij} Q_j) = 0 \quad (3-36)$$

so we get:

$$P_i = \frac{1}{\alpha_{ii}} \left[\beta_{ii} Q_i - \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij} P_j - \beta_{ij} Q_j) \right] \quad (3-37)$$

In this formula, we know that the P_i is the real power injection at node i . It does not represent the demand power or the generation input power. But, it has a relationship with the two. That is :

$$P_i = P_{DG_i} - P_{D_i} \quad (3-38)$$

Where the P_{DG_i} is the real power injection from DG placed at node i , it is the input real power of the node. The P_{D_i} is the demand load of the node i . To put all the equations above here, we could get:

$$P_{DG_i} = P_{D_i} + \frac{1}{\alpha_{ii}} [\beta_{ii} Q_i - \sum_{j=1, j \neq i}^N (\alpha_{ij} P_j - \beta_{ij} Q_j)] \quad (3-39)$$

When we get this formula, we could easily find that only P_{DG_i} in the formula is uncertain variable for each bus. And as we know the meaning of the equation is to get the minimize size of P_i for minimizing the Power loss. So the equation here which could give us the Distributed Generator result is the equation that give us the optimized Distributed Generator size. Beside this size of the Distributed Generator, all the other sizes which is being put in the bus will lead to a higher power loss in the whole system.

Here, when we install the DG into the optimal bus i in the system. because of node equation: $P_i = P_{DG_i} - P_{D_i}$ (3-38), we know that the P_i must be changed. This will result to the change of voltage and angle in bus i . And then , all the voltages and angle will changed after running the base flow. At last the coefficient α and β will change which may involve the value of the power loss. In the paper[], author explain that

numerical result shows the accuracy gained in the size of DG by updating α and β is too small to involve power loss result. In this way, we could get the optimized size and location for the DG by easily avoid recalculating the power flow again. Then, as the same theory, we could easily get the optimized power loss. The following is the computational procedure for this method:

Step1: Sep up the modeling with power world simulator:

Step2: Run the base power flow.

Step3: Get the optimal size for the DG in every bus in the system with Eq(3-39).

Step4: Calculate the loss using Eq(3-16) on each bus which has being placing by optimized DG(we get this DG in step3).

Step5: Confirm the bus which make the power loss minimum while the DG is placed. Here is the optimum location for DG.

step6: Run the load flow with DG to get the final result.

The algorithm figure for the method is at the end of this chapter:

3.4.2 Method 2 start from analysis the real DG input power:

In this method, we may need to step back the power loss equation:

$$P_L = \sum_{k=1}^n [\alpha_{jk}(P_j P_k + Q_j Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k)] \quad (3-39)$$

Now , as we start to consider the DG real power earlier. we may need to consider the relationship between the real power part and reactive power part from DG[43], let us first assuming:

$$a = (\text{sign}) \tan(\cos^{-1}(\text{PF}_{\text{DG}})) \quad (3-40)$$

so that we get:

$$Q_{\text{DGi}} = aP_{\text{DGi}} \quad (3-41)$$

here we need to know:

sign=+1, DG injecting Q;

sign=-1: DG consuming Q;

then, as we know the relationship on the node is:

$$P_i = P_{\text{DGi}} - P_{\text{Di}}$$

$$Q_i = Q_{\text{DGi}} - Q_{\text{Di}} = aP_{\text{DGi}} - Q_{\text{Di}} \quad (3-41)$$

Here, we take the formulas above (3-41), back to the power loss equation, then

we could get :

$$P_L = \sum_{i=1}^N \sum_{j=1}^N [\alpha_{ij}[(P_{\text{DGi}} - P_{\text{Di}})P_j + (aP_{\text{DGi}} - Q_{\text{Di}})Q_j] + \beta_{ij}[(aP_{\text{DGi}} - Q_{\text{Di}})P_j - (P_{\text{DGi}} - P_{\text{Di}})Q_j]] \quad (3-42)$$

Now, we are going to do the partial derivative of the above equation. Here we do the derivative of DG, P_{DG} ;

$$\frac{\partial P_L}{\partial P_{\text{DGi}}} = 2 \sum_{j=1}^N [\alpha_{ij}(P_j + aQ_j) + \beta_{ij}(aP_j - Q_j)] = 0 \quad (3-43)$$

then we take Eq(3-39) in so that we get:

$$\alpha_{ii}(P_i + aQ_i) + \beta_{ii}(aP_i - Q_i) + \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij}P_j - \beta_{ij}Q_j) + a \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij}Q_j + \beta_{ij}P_j) = 0$$

(3-44)

It followed by:

$$X_i = \sum_{\substack{j=1 \\ j \neq i}}^n (\alpha_{ij}P_j - \beta_{ij}Q_j)$$

$$Y_i = \sum_{\substack{j=1 \\ j \neq i}}^n (\alpha_{ij}Q_j + \beta_{ij}P_j) \quad (3-45)$$

here, we put the equation(3-41)(3-42)(3-43)(3-44)(3-45) together, then, we get:

$$\alpha_{ii}(P_{DG_i} - P_{Di} + a^2P_{DG_i} - aQ_{Di}) + \beta_{ii}(Q_{Di} - aP_{Di}) + X_i + aY_i = 0 \quad (346)$$

As we get the above equation, we could find out optimal size of DG at each bus i to minimizing loss :

$$P_{DG_i} = \frac{\alpha_{ii}(P_{Di} + aQ_{Di}) + \beta_{ii}(aP_{Di} - Q_{Di}) - X_i - aY_i}{a^2\alpha_{ii} + \alpha_{ii}} \quad (3-47)$$

In the above equation(3-47), we find the "a" is the variable that we need to confirm in order to get the P_{DG} result. The following is the table that hold the different situations for a and Power factor (PF_{DG}):

	PF_{DG}	a	Sign	Equation
Type 1	1	0	N/A	$P_{DG_i} = P_{Di} - \frac{1}{\alpha_{ii}} [\beta_{ii}Q_{Di} + \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij}P_j - \beta_{ij}Q_j)]$
Type 2	0	∞	N/A	$Q_{DG_i} = Q_{Di} + \frac{1}{\alpha_{ii}} [\beta_{ii}P_{Di} - \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij}Q_j + \beta_{ij}P_j)]$

Type 3	(0,1)	c	+1	$P_{DGi} = \frac{\alpha_{ii}(P_{Di} + aQ_{Di}) + \beta_{ii}(aP_{Di} - Q_{Di}) - X_i - aY_i}{a^2\alpha_{ii} + \alpha_{ii}}$ $Q_{DGi} = aP_{DGi}$
Type 4	(0,1)	c	-1	

Table 4

Now, as we are trying to list out a common situation not a special one, we need to confirm the a as a common situation. in the paper[], the author give us an explanation that the power factor of demand here is almost the same as DG when we want to get the minimized total power loss. So if we do:

$$P_D = \sum_{i=1}^N P_{Di}$$

$$Q_D = \sum_{i=1}^N Q_{Di} \quad (3-48)$$

We then, get the following:

$$PF_{DG} = PF_D \quad (3-49)$$

when , we step back to the node relationship equation it is easy to get this result only because when the p.f of demand are the same the p.f of DG, the input could be transmit into the demand at the maximum way, which means the total loss is minimum.

Then, as the same consideration of the former method. We could again, get the P_{DGi} . And then, we could also get the total power loss after optimized.

Here we also list out the computational procedure

Step1: Step up the modeling with power flow:

Step2: Run the base power flow.

Step3: Calculate the Power Factor with Eq(3-48) and Eq(3-49)

Step4: Get the optimal size for the DG in every bus in the system with Eq(3-47).

Step5: Calculate the loss using Eq() on each bus which has being placing by optimized DG(we get this DG in step3).

Step6: Confirm the bus which make the power loss minimum while the DG is placed. Here is the optimum location for DG.

Step7: Run the load flow with DG to get the final result

3.4.3 Comparison of the two methods:

Here , we use two methods to analysis the result for getting the optimal DG power. We could find out that there are two main difference in the two method:

1. When we do the partial derivative, method one do it by inject power P_i while the second method do it by DG power P_{DG} .
2. The second consider more about the p.f. factor problem. It consider all the possible for the PF.

We will compare the result of the two methods in Chapter 5 to see if they have relationship or some majority difference.

Chapter 4 Methods and tools proposed in the paper:

In the paper, we start with improve of analysis (IA) method. We make some improve on the original formula for power loss. The aim of this to set up the relationship between the power loss and base power flow data. In this way, we only need to run the power flow and the data from the bus part of system(not the whole system) if we hope to get the power loss. Comparing with if we take consideration of the whole system. This way is much more easier. Then, when we set the analysis modeling, we get the optimized size and location by the modeling. In realize the optimization process, we use Power World simulator to get the power flow result. Then, we compute the optimization of location and size of the Distributed generation(DG) by Matlab. We write a code for the optimization and use the result of by Power World to get the DG location and size. Then, we take the result back to IA modeling and use Power World to calculation the optimized power loss.

So, In this chapter, we list out the major method and tools that had been put into the research process. And then, give them a brief analysis.

4.1 IA method:

In the chapter2, we give IA a brief explanation. Improve of analysis called IA is a new method to analysis the problem. Unlike the traditional optimization method, it does not list out all the objective function and possible constraints. It start with a deeper analysis of the optimal process, and try to get a clearer and direct relationship between the uncertain varieties and give data and information. In this way, we could get the calculation process faster.

In this paper, we start but did not follow with the original power relationship as follow (4-1):

$$P_i + jQ_i = \sum_{i=1}^n S_i = \sum_{i=1}^n V_i J_i^* \quad (4-1)$$

Then, we take the matrix calculation (4-2):

$$\begin{aligned} (A + B)^T &= A^T + B^T \\ (AB)^T &= B^T A^T \\ (A^T)^T &= A \\ Z_{bus} &\triangleq R + jX = \begin{bmatrix} r_{11} & \dots & r_{1n} \\ & & \\ r_{n1} & & r_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & \dots & x_{1n} \\ & & \\ x_{n1} & & x_{nn} \end{bmatrix} \end{aligned} \quad (4-2)$$

and vector calculation (4-3),

$$P_i + jQ_i = J_{bus}^T Z_{bus}^T J_{bus}^* = J_{bus}^T Z_{bus} J_{bus}^* \quad (4-3)$$

And then, use an abbreviate method to reduce the induced formula to get the following power loss equation:

$$\frac{\partial P_L}{\partial P_{Gi}} = \sum_{k=1}^n \frac{\partial}{\partial P_i} [\alpha_{jk}(P_j P_k + Q_j Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k)]$$

Then, in the two method they all use partial derivative to get the optimal size for DG. No matter where they start to calculate, they are all based on an analysis about the derivative. Here, we use a derivative method to avoid listing out too much useless constraints which may lead us to a much more complicated calculation process.[5][42]

4.2 Power World simulator for power flow:

As the introduction above, we know the usage of Power World simulator is to get the base power flow data. We introduced the Power World in chapter 2.

Power World simulator is a software that specialist for the power system calculation. And it is really fast and convenient to compute the power flow. Comparing with using Matlab code to calculate the power flow, it is really a better choice. Because when you use the Matlab code. [44]You need to spend a lot of time on writing the code and then to make sure the logic of the code is right. After you finish all of these, you may still not be able to get the good result. Then, you need to check many "spot" such as input data, angle expression, step consequence and so on. In the Power world, what you only need to is to set up the modeling of your system. And then, you only need to input all the starting data for the power system(they are Voltage, angel of voltage, real power and reactive power for the demand and

impedance). Although, we have to say, we need spend some time on set up the modeling, it is much more faster than writing a code for calculating the power flow.

In this research, the power world give many help while doing the power flow calculation. First, when we input the generator as follow (fig 4-1):[45][46]

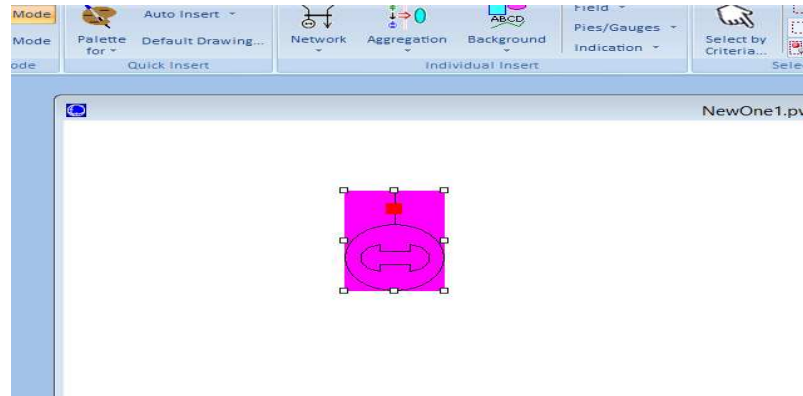


Fig 4-1 Generator input

the system will reminder you (Figure 4-2)

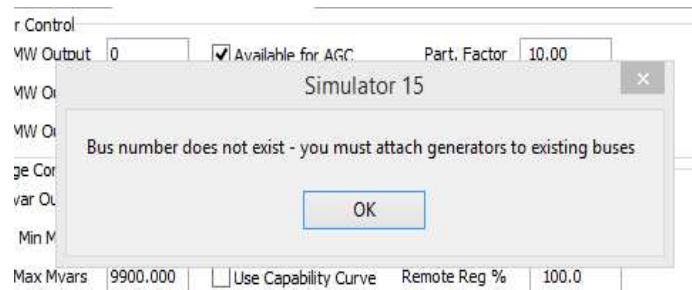


Fig 4-2 System reminder

This is remind you that you need to add bus first because if you do not have bus there is no place to lay the generator on. This is a simple problem. But it proves that the window and operator of the power world simulator is friendly. It will help to check the step and remind you problem(you could see that the generator without bus to place is red).

Then, you start to put the bus on as follow (figure 4-3).

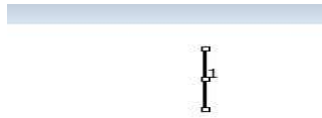


Fig 4-3 Bus figure in Power World

In this way, you have to choose the orientation of the bus like (figure 4-4):

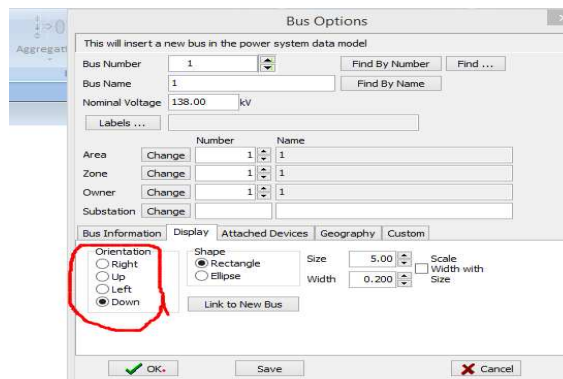


Fig 4-4 Bus information

then, you input all the data easily as introduced in the chapter 2.

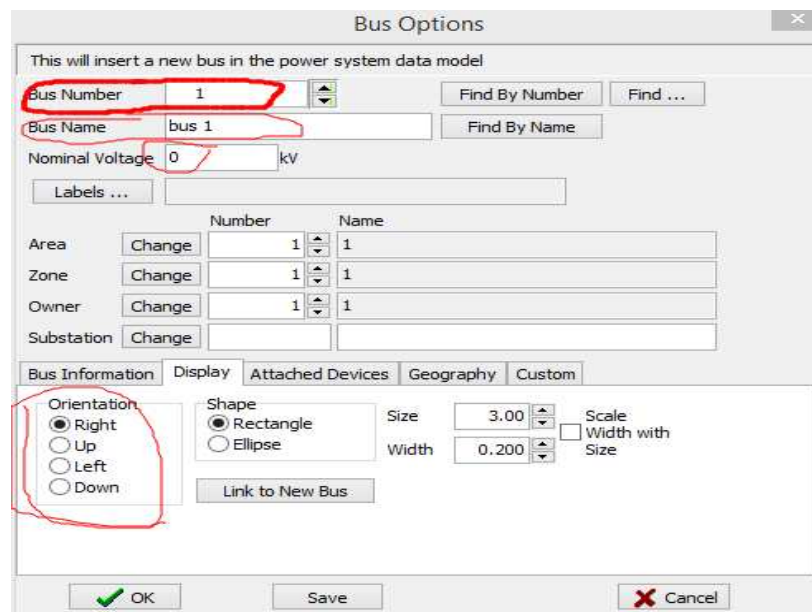


Fig 4-5 Bus Option

and also (figure 4-6):

Bus Options

This will insert a new bus in the power system data model

Bus Number: 1 Find By Number Find ...

Bus Name: bus 1 Find By Name

Nominal Voltage: 0 kV

Labels ...

	Number	Name
Area	1	1
Zone	1	1
Owner	1	1
Substation		

Bus Information Display Attached Devices Geography Custom

Bus Voltage

Voltage (p.u.) 1.00

Angle (degrees) 0.00

☐ System Slack Bus

OK Save Cancel

Fig 4-6 Bus Option

If the bus1 is slack bus , you need to pay attention to the blanks i quote in red line.

After these, we need to add the generator to the slack bus (4-7):

Generator Options

Bus Number: 1
 Bus Name: bus 1
 ID: 1
 Area Name: 1
 Labels ...

Find By Number
 Find By Name
 Find ...

Status:
☐ Open
☒ Closed
 Generator MVA Base: 100.00

Fuel Type: Unknown
 Unit Type: UN (Unknown)

Display Information | **Power and Voltage Control** | Costs | Fault Parameters | Owner, Area, Zone, Sub | Custom

Power Control
 MW Output: 0
 Min. MW Output: 0.000
 Max. MW Output: 1000.000
☒ Available for AGC
 Part. Factor: 10.00
☒ Enforce MW Limits

Voltage Control
 Mvar Output: 0.000
 Min Mvars: -9900.000
 Max Mvars: 9900.000
☒ Available for AVR
 Regulated Bus Number: 1
 SetPoint Voltage: 1.0000
☐ Use Capability Curve
 Remote Reg %: 100.0

Wind Control Mode
 Mode: None
 Power Factor: 1.0000

MW									
Min Mvar									
Max Mvar									

OK Save Cancel Help

Fig 4-7 Generator Option

Here, we do not need to a starting data for the generator power. we could give an 0. Because we assume that the generator have a very big capacity that could hold whatever we out in the system. So, we only need to do is to give a wide constraint of the power(second red circle). So does the voltage.

And also as (figure 4-8):

Generator Options

Bus Number: 1 Status: ☐ Open ☒ Closed Generator MVA Base: 100.00

Bus Name: bus 1 Find ...

ID: 1

Area Name: 1 Fuel Type: Unknown

Unit Type: UN (Unknown)

Labels ...

Display Information | Power and Voltage Control | Costs | Fault Parameters | Owner, Area, Zone, Sub | Custom

Display Size: 10.0 ☒ Scale Width with Size

Display Width: 7.50

Pixel Thickness: 1

☒ Anchored

Orientation: ☐ Right ☐ Up ☒ Left ☐ Down

Rotor Shape: Dog Bone

☐ Fill Rotor Symbol with Color 2

Fill with Color 2:

Fig 4-8 Information Table

So we get (figure 4-9):

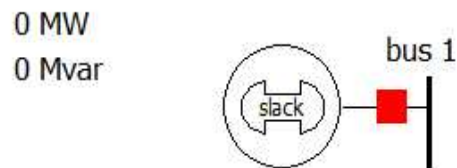


Fig 4-9 Generator figure with information in Power World

Here, we note that when you install the generator in the right way, the generator do not have colors.

Before we put transmission line into the system , we also need to put bus2 in to the system:

Bus Options

This will insert a new bus in the power system data model

Bus Number: 2 Find By Number Find ...

Bus Name: bus 2 Find By Name

Nominal Voltage: 0.09634 kV

Labels ...

	Number	Name
Area	1	1
Zone	1	1
Owner	1	1
Substation		

Bus Information Display Attached Devices Geography Custom

Orientation: ☒ Right ☐ Up ☐ Left ☐ Down

Shape: ☒ Rectangle ☐ Ellipse

Size: 3.00 Width: 0.200 Scale Width with Size

Link to New Bus

OK Save Cancel

Fig 4-10 Bus option revise

Here, we need to put the data of the voltage in to the bus(here, it is bus2).

In this way we could put the bus2 on as follow (figure 4-11):

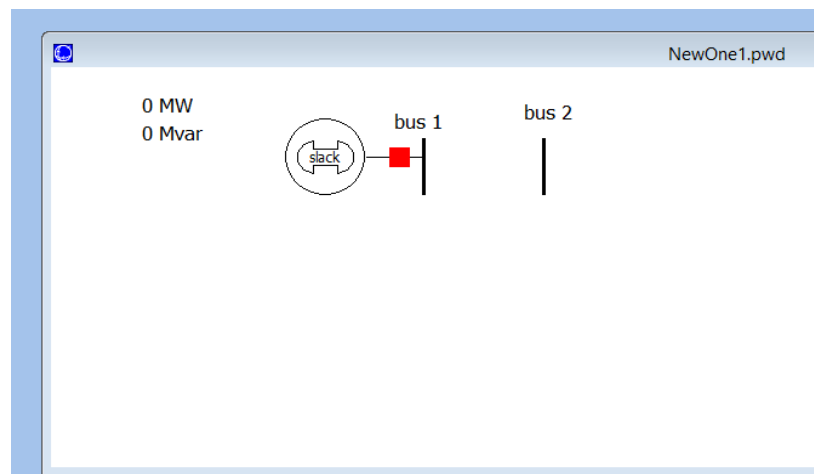


Fig 4-11 Set up the modeling

Following this, we could put the transmission line on to the system. We put it between the bus1 and bus2, as follow (figure 4-12):

Branch Options

Line Number: 1, Name: bus 1, Area Name: 1 (1), Nominal kV: 12.7

To Bus: 2, Name: bus 2, Area Name: 1 (1), Nominal kV: 0.1

Labels: no labels

Find By Numbers, Find By Names, Find ...

☒ From End Metered

☒ Default Owner (Same as From Bus)

Display Parameters Fault Info Owner, Area, Zone, Sub Custom Stability

Status: ☐ Open, ☒ Closed

Branch Device Type: Line

Allow Consolidation: ☐

Length: 0.00

Calculate Impedances >

Convert Line to Transformer

Per Unit Impedance Parameters:

Series Resistance (R)	0.09220000
Series Reactance (X)	0.04700000
Shunt Charging (B)	0.00000
Shunt Conductance (G)	0.00000

Has Line Shunts: ☐ Line Shunts

MVA Limits:

Limit A	0.000
Limit B	0.000
Limit C	0.000
Limit D	0.000
Limit E	0.000
Limit F	0.000
Limit G	0.000
Limit H	0.000

OK Save Cancel Help

Fig 4-12 Branch Option table

Here, we need to put the data for the bus into the line information (figure 4-13):

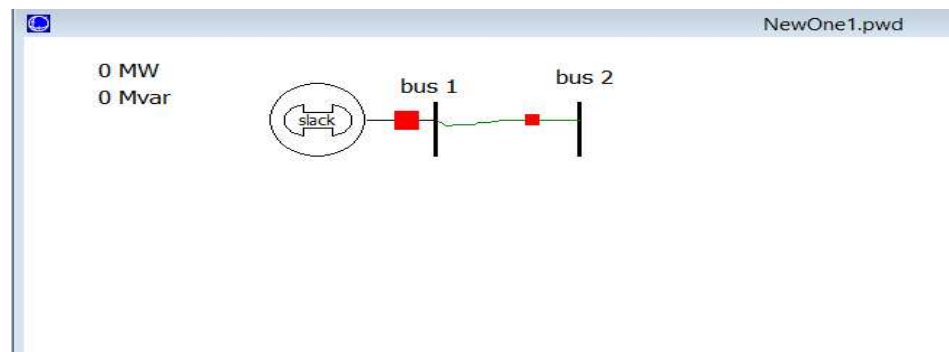


Fig 4-13 Draw the modeling

At last, we need to put a load on bus2 (figure 4-14)

The screenshot shows the 'Load Options' dialog box. The 'Bus Number' is 2 and 'Bus Name' is 'bus 2'. The 'Load Information' tab is selected, showing 'Constant Power' with 'MW Value' at 0.100 and 'Mvar Value' at 0.09000. The 'Display Information' section shows 'Display Size' at 6.00 and 'Display Width' at 0.80. The 'Status' is set to 'Closed'.

Fig 4-14 Load Option revise

Here, we only need to complete the information in Load information part. we give the size of power and the display. But, in some case that need us to calculate the optimize power flow with the economic dispatch, we need to open the blank "OPF load Dispatch" at the right of "load information".

Then, we get the following figure (figure 4-15)

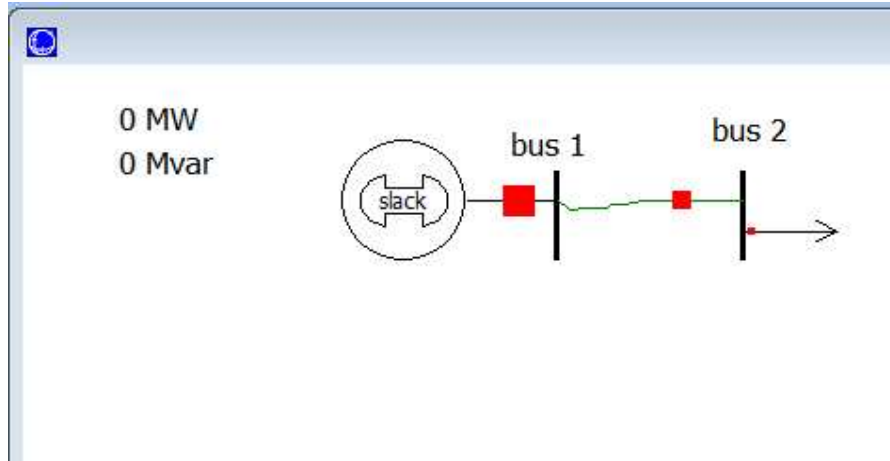


Fig 4-15 Draw some basic part of modeling

When, we finish this, we could then continue to add other 31 buses, 31 transmission lines and 31 load to get the 33 bus and 38 bus (In)

4.3 Matlab for optimal modeling:

For doing the optimization, we choose the Matlab. We edit a paragraph of code to realize the optimization. It indeed take us some time to edit, to address , to get the result. It is fastest we could try to do the optimization right now. However, its calculation function is great and the result will be list out clearly.[47][48]

There is not too much to say on the Matlab. However, when we talk about optimization we could say something about Matlab. When we think about doing the calculation for optimization[49].he Matlab simulator has an unique advantage. When we are dueling with a optimization problem, no matter you are doing the traditional optimal problem(like objective and constraints) or new method for optimal(like IA

method), calculation is always a problem[50]. The Matlab simulator could put all the data and equations into a huge matrix. And then, Matlab would use its great calculation ability for matrix to do the optimization problem. It is the best to use Matlab to deal with the optimization problem. [51]

Chapter 5: Result and analysis:

In this chapter, we first use a 33 P-Q bus modeling to make a comparison for method 1 and method 2 to find a better method. Then, we use to the better method to get the result for another 16 test bus modeling (but P-V bus).

5.1 Result for method 1:

The following Figure, figure 5-1 is a 33 bus modeling built in the power world.

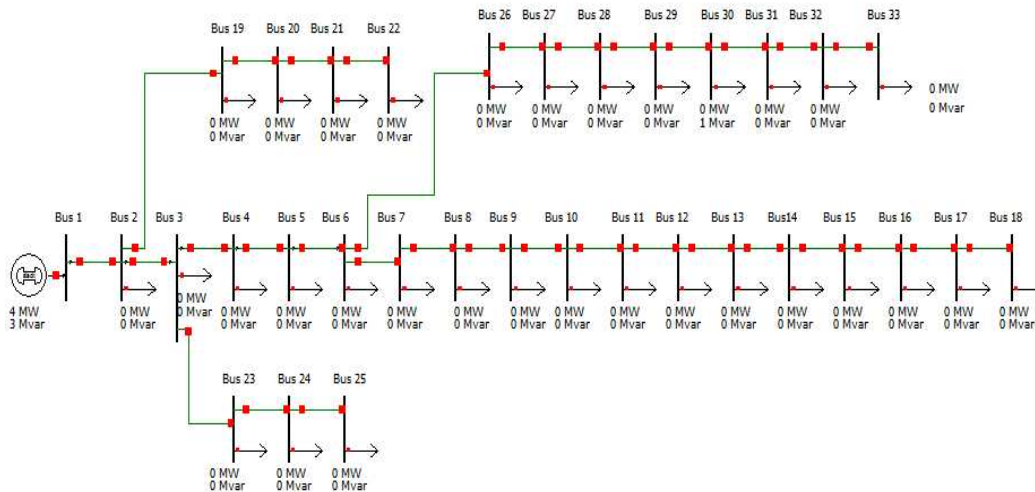


Fig 5-1 The modeling for 33 bus

Case Summary for Current Case

Number of Devices in Case			
Buses	33	Series Capacitors	0
Generators	1	LTCs (Control Volt)	0
Loads	32	Phase Shifters	0
Switched Shunts	0	Mvar Controlling	0
Trans. Lines (AC)	32	Breakers	0
2 Term. DC Lines	0	Disconnects	0
Multi-Term. DC	0	ZBRs	0
Areas	1	Islands	1
Zones	1	Interfaces	0
Substations	0	Injection Groups	0

Case pathname: test.sys.PWB

Case Totals (for in-service devices only)

	MW	Mvar
Load	3.7	2.3
Generation	4.1	2.5
Shunts	0.0	0.0
Losses	0.4	0.3

Generator Spinning Reserves

Positive [MW]	Negative [MW]
995.9	4.1

Negative MW Loads and Generators

	MW	Mvar
Load	0.0	0.0
Generation	0.0	0.0

Slack Buses:

Bus 1 (1); in Area 1 (1)

Print ? Help Close

Fig 5-2 The original result for 33 bus

By using the modeling in this figure, we find the calculation for power flow is at the minimize size of 1 MW(This is why the real and reactive power on the buses are all keep on zero). As we run the Power World with the model, we could get the total power loss is 0.5 MW with DG

5.1.1 For method 1:

Then, we input the data to Matlab code for optimal the location and size for DG(the data is gain from 33 bus modeling after the power flow calculation above).

Then , we get the follow result, figure5-3 and figure 5-4, as follow:

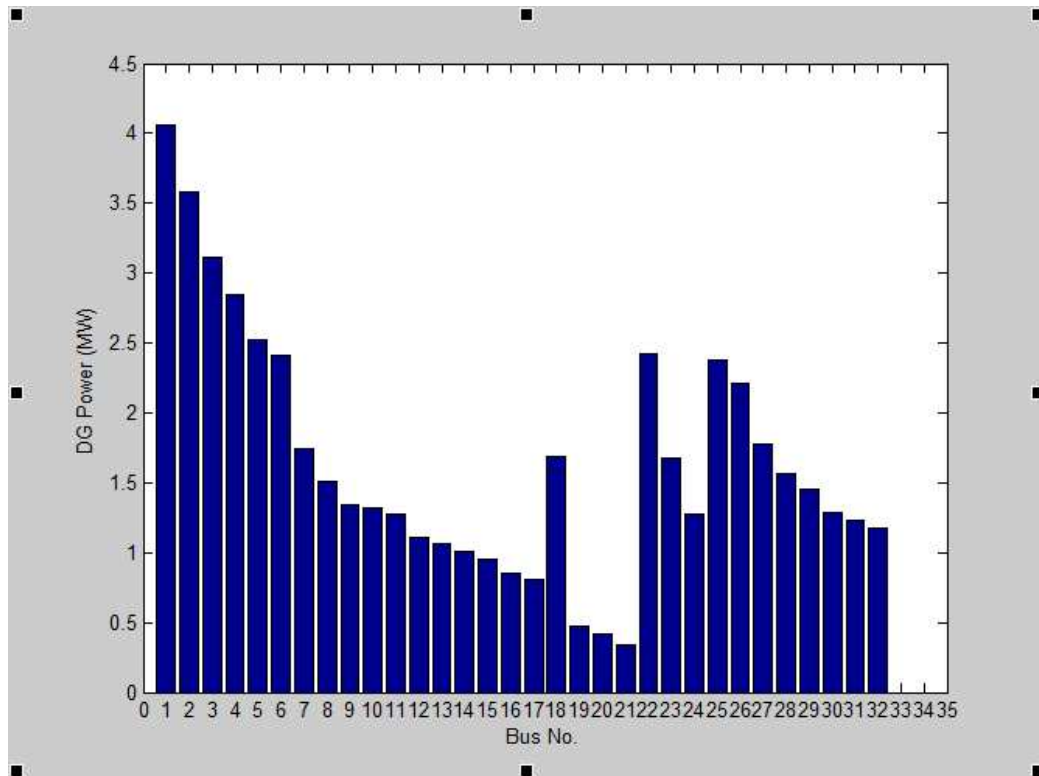


Fig 5-3 Result for method1 on relation by bus No. and P_{DG}

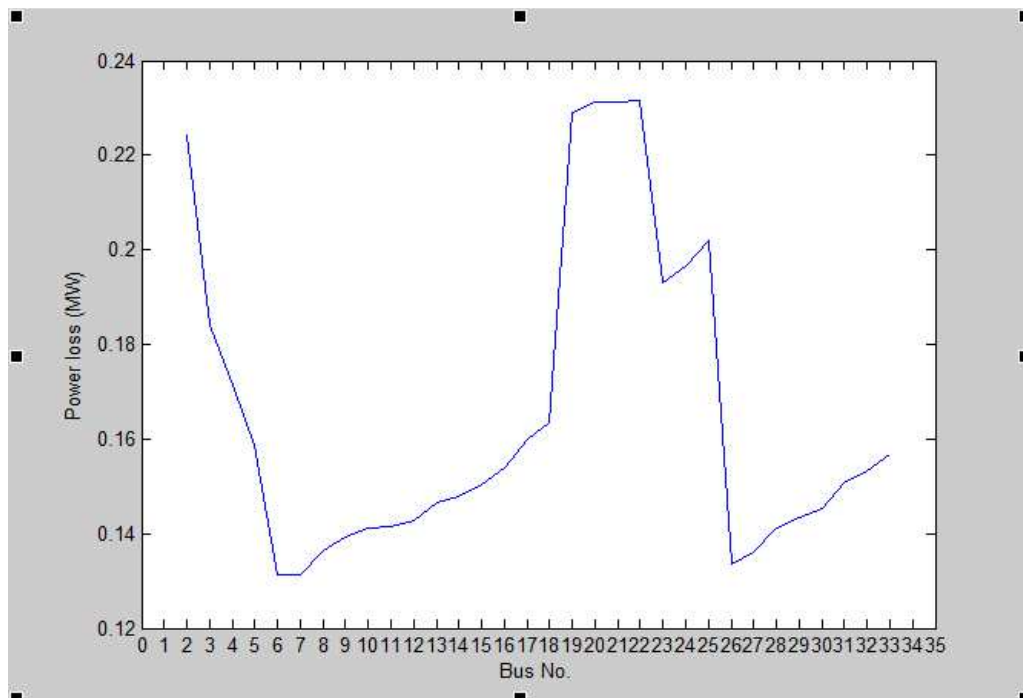


Fig 5-4 Result for method1 on relation by P_{DG} and P_L

In the figure 5-4, we could see that the optimal DG location should be in bus6 or bus7. Then, we mark the bus 6 and bus 7 as follow figure 5-5, 5-6:

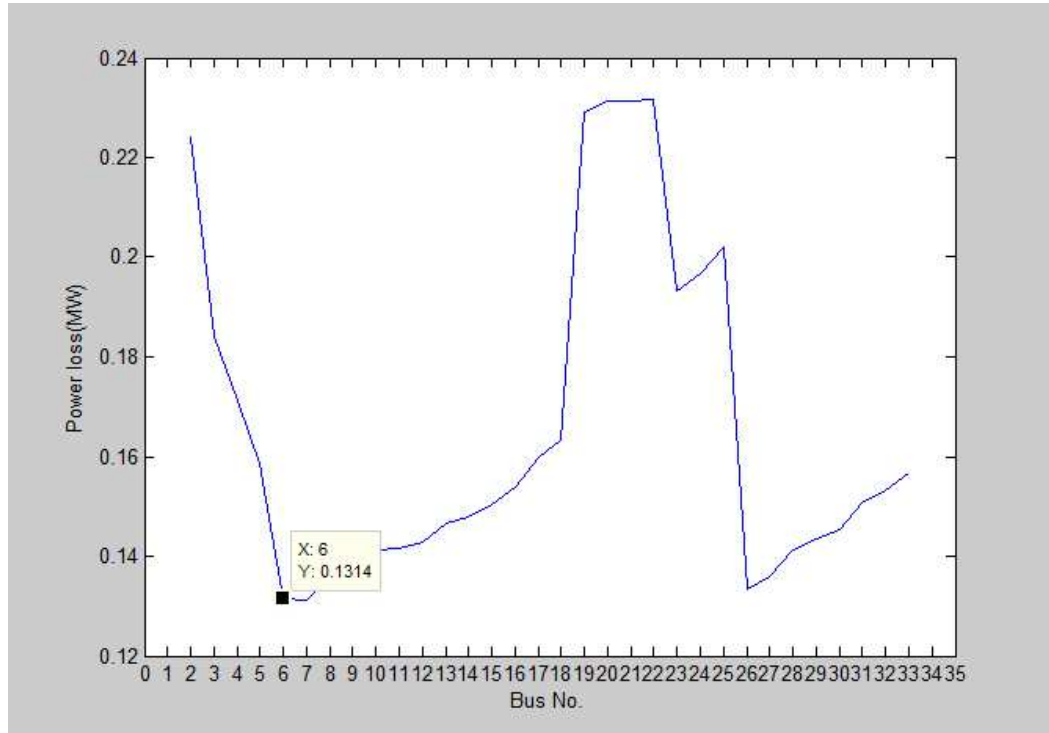


Fig 5-5 The power loss on bus 6 for method 1

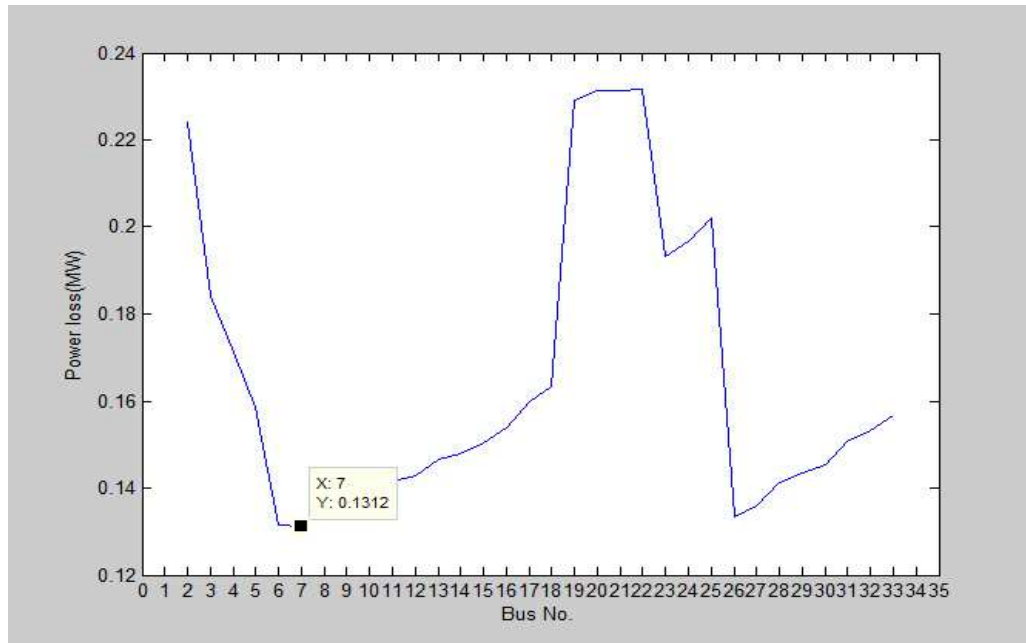


Fig 5-6 The power loss on bus 7 for method 1

Here, we get the approximate result of power loss after placing in bus 6 and bus 7, they are: $P_{L6}=0.01314\text{MW}$, $P_{L7}=0.01312\text{MW}$; As the result are so similar, we place the optimal DG back in bus 6 and bus7 with their own optimal DG size in Figure 5-1. In this case we get the figure as follow figure 5-7 and 5-8:

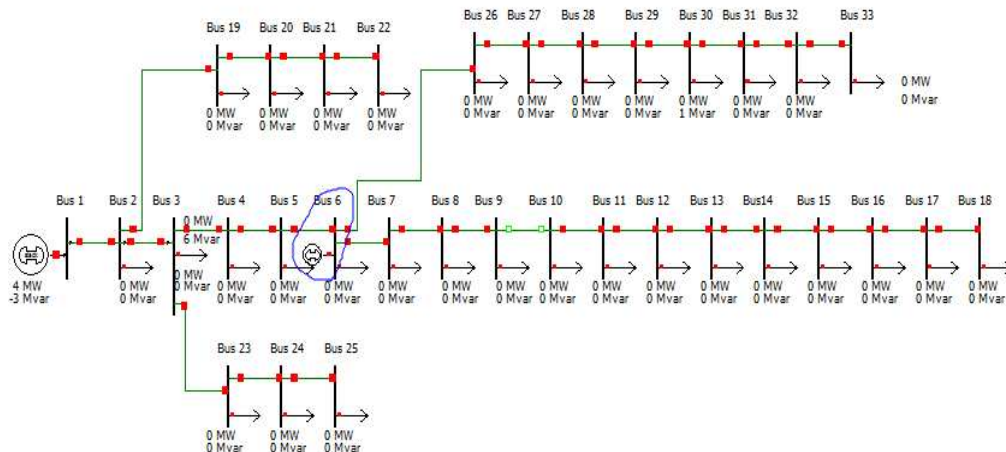


Fig 5-7 Optimal DG put on bus 6.

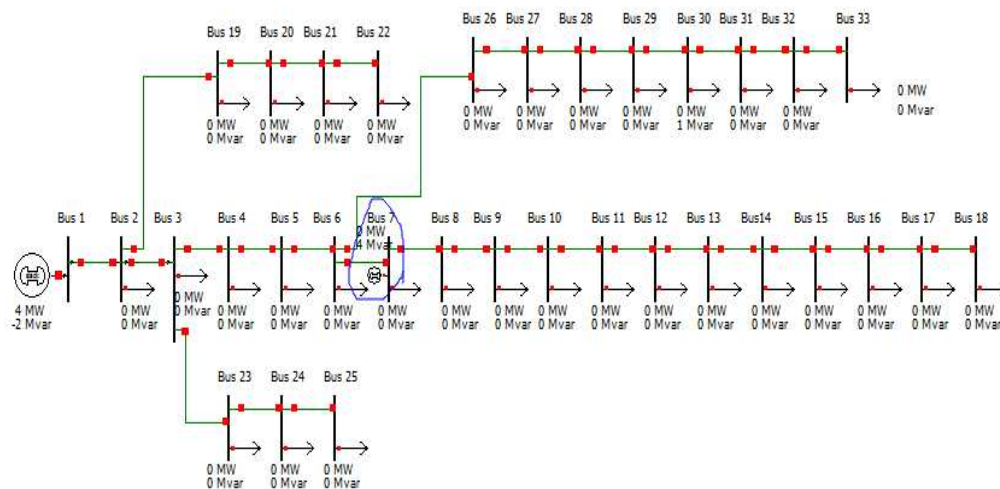


Fig 5-8 Optimal DG put on bus 7.

After running the modeling in figure 5-7 and figure 5-8, we could finally get the power loss result they are in figure 5-9 and 5-10:

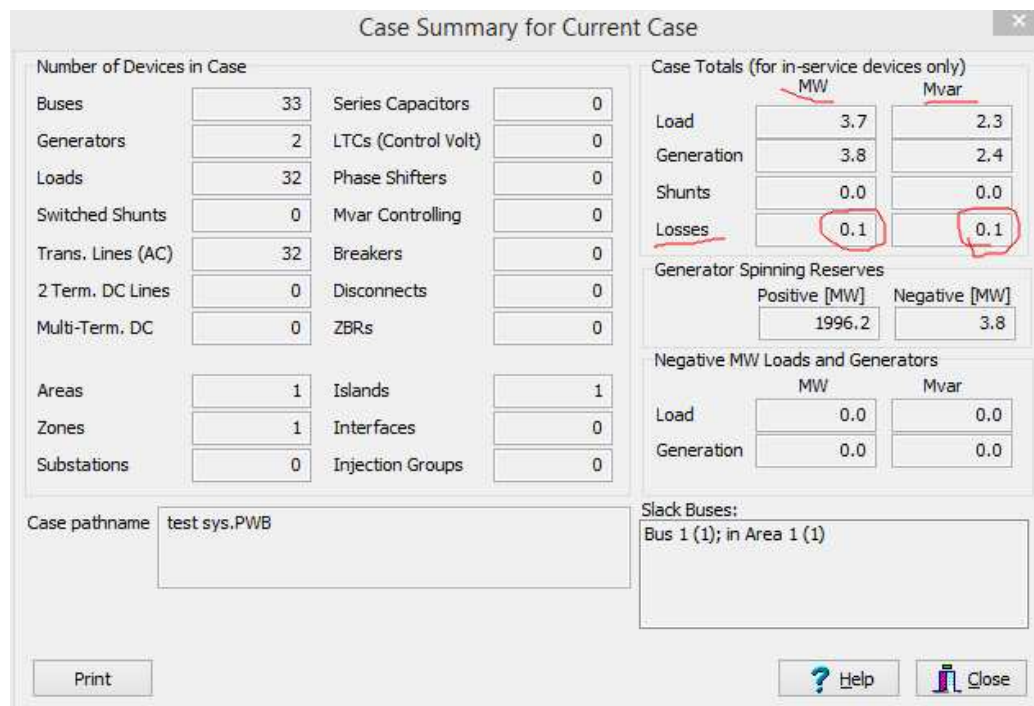


Fig 5-9 The Power Loss when put DG on bus 6 for method 1

Case Summary for Current Case			
Number of Devices in Case			
Buses	33	Series Capacitors	0
Generators	2	LTCs (Control Volt)	0
Loads	32	Phase Shifters	0
Switched Shunts	0	Mvar Controlling	0
Trans. Lines (AC)	32	Breakers	0
2 Term. DC Lines	0	Disconnects	0
Multi-Term. DC	0	ZBRs	0
Areas	1	Islands	2
Zones	1	Interfaces	0
Substations	0	Injection Groups	0
Case pathname: test sys1.pwb			
Case Totals (for in-service devices only)			
	MW	Mvar	
Load	3.3	2.1	
Generation	3.4	2.2	
Shunts	0.0	0.0	
Losses	0.1	0.1	
Generator Spinning Reserves			
	Positive [MW]	Negative [MW]	
	1996.6	3.4	
Negative MW Loads and Generators			
	MW	Mvar	
Load	0.0	0.0	
Generation	0.0	0.0	
Slack Buses:			
Bus 1 (1); in Area 1 (1)			

Fig 5-10 The Power Loss when put DG on bus 7 for method 1

Here, we could get total power loss after putting in the optimized DG. As we get :

$$s = \sqrt{P^2 + Q^2}$$

So, here, for bus 6, it is 0.12MW, for bus 7 it is 0.11MW(The result here is shown in power world detail result sheet.)

5.1.2 For method 2:

In the method 2, we firstly use the modeling in figure 5-1 again to get the result for the power flow data. Then, we input the data to the Matlab code for the method 2. Here, we get the following result figure 5-11 and 5-12:

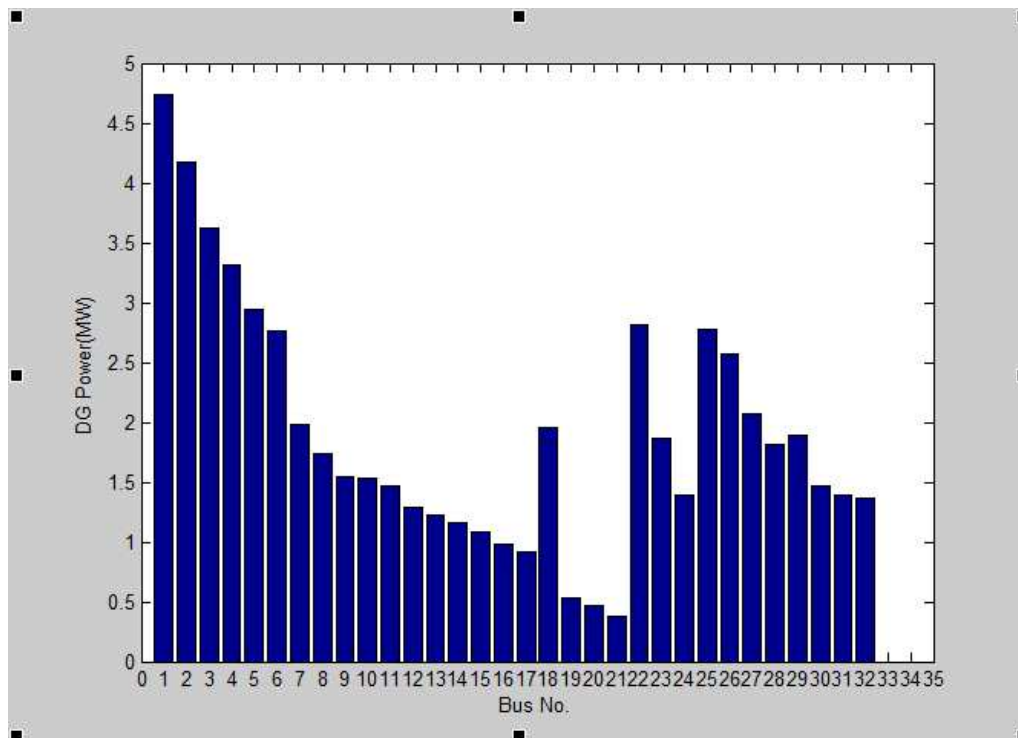


Fig 5-11 Result for method2 on relation by bus No. and P_{DG}

Case Summary for Current Case

Number of Devices in Case				Case Totals (for in-service devices only)			
		MW	Mvar			MW	Mvar
Buses	33	Series Capacitors	0	Load	3.7		2.3
Generators	1	LTCs (Control Volt)	0	Generation	4.1		2.5
Loads	32	Phase Shifters	0	Shunts	0.0		0.0
Switched Shunts	0	Mvar Controlling	0	Losses	0.4		0.3
Trans. Lines (AC)	32	Breakers	0	Generator Spinning Reserves			
2 Term. DC Lines	0	Disconnects	0	Positive [MW]	995.9	Negative [MW]	4.1
Multi-Term. DC	0	ZBRs	0	Negative MW Loads and Generators			
Areas	1	Islands	1	Load	0.0		0.0
Zones	1	Interfaces	0	Generation	0.0		0.0
Substations	0	Injection Groups	0	Slack Buses:			
Case pathname: test sys.PWB				Bus 1 (1); in Area 1 (1)			

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Fig 5-12 Result for method2 on relation by bus No. and P_{DG}

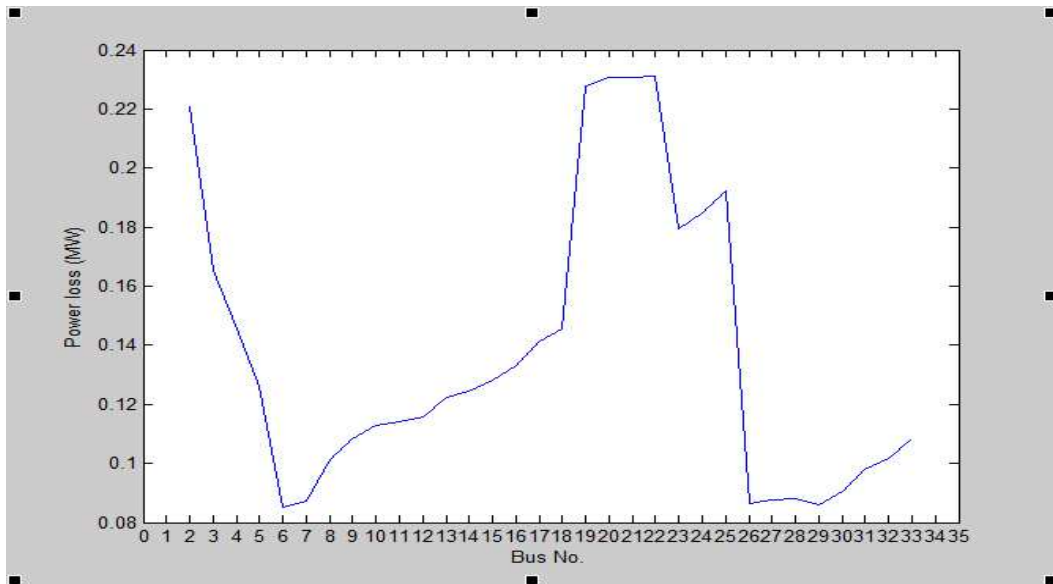


Fig 5-13 Result for method 2 on relation by P_{DG} and P_L

Here again, we get the approximate result of power loss after placing in bus 6 and bus 7 again, they are: $P_{L6}=0.008521\text{MW}$, $P_{L7}=0.008750\text{MW}$; As the result are again similar to each other, we now settle down the optimal DG back in bus 6 and bus 7 with their own optimal DG size in Figure 5-1. In this case we get the figure as follow figure 5-13 and 5-14:

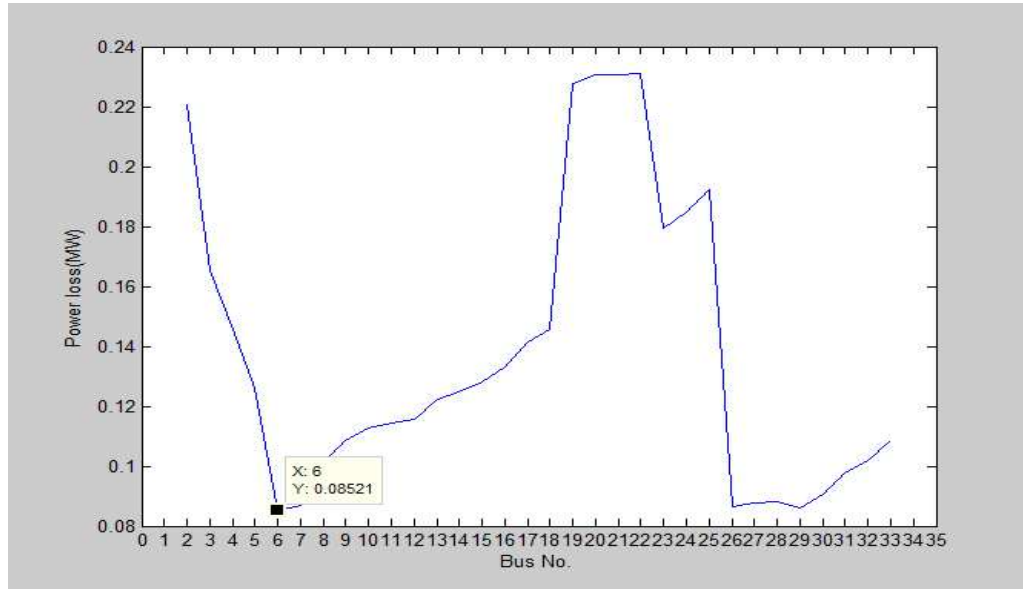


Fig 5-13 The power loss on bus 6 for method 2

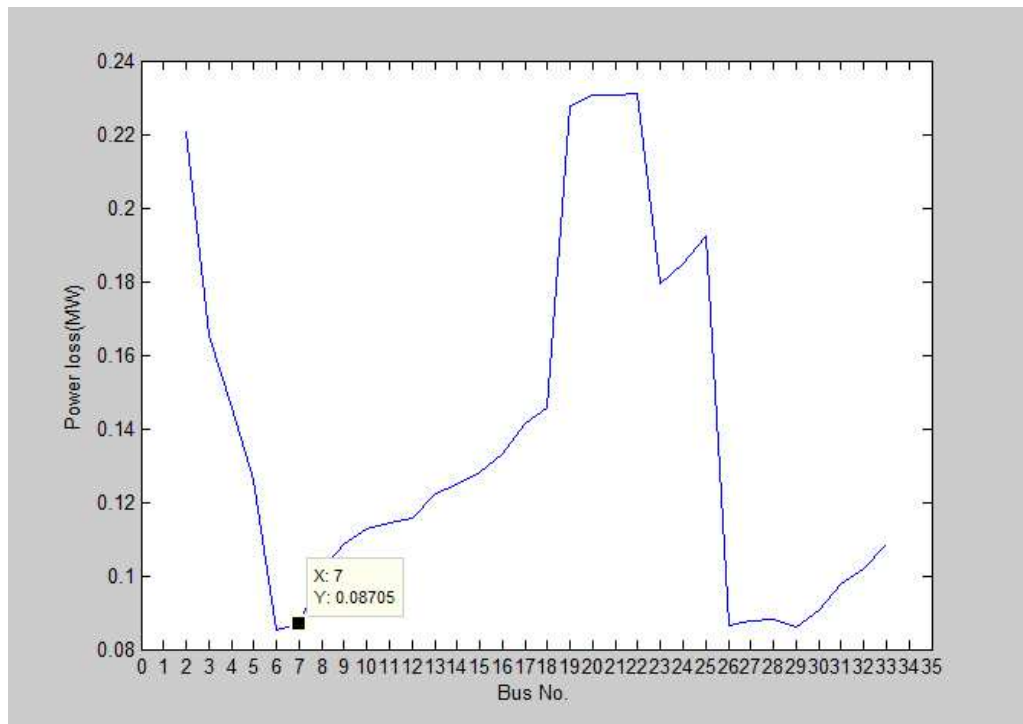


Fig 5-14 The power loss on bus 7 for method 2

After get this figure, we still want to put the optimized DG back to the bus 6 and bus 7, the modeling for these two are exactly the same as in figure 5-7 and 5-8.

By doing these, we could then run the modeling as same as in figure 5-6 and 5-8. The follow two figure is the result of simulating for exactly power loss.

Case Summary for Current Case

Number of Devices in Case			
Buses	33	Series Capacitors	0
Generators	2	LTCs (Control Volt)	0
Loads	32	Phase Shifters	0
Switched Shunts	0	Mvar Controlling	0
Trans. Lines (AC)	32	Breakers	0
2 Term. DC Lines	0	Disconnects	0
Multi-Term. DC	0	ZBRs	0
Areas	1	Islands	1
Zones	1	Interfaces	0
Substations	0	Injection Groups	0

Case pathname: test sys,PWB

Case Totals (for in-service devices only)		
	MW	Mvar
Load	3.7	2.3
Generation	3.8	2.4
Shunts	0.0	0.0
Losses	0.1	0.1

Generator Spinning Reserves	
Positive [MW]	Negative [MW]
1996.2	3.8

Negative MW Loads and Generators		
	MW	Mvar
Load	0.0	0.0
Generation	0.0	0.0

Slack Buses:
Bus 1 (1); in Area 1 (1)

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Fig 5-15 The Power Loss when put DG on bus 6 for method 2

Case Summary for Current Case

Number of Devices in Case			
Buses	33	Series Capacitors	0
Generators	2	LTCs (Control Volt)	0
Loads	32	Phase Shifters	0
Switched Shunts	0	Mvar Controlling	0
Trans. Lines (AC)	32	Breakers	0
2 Term. DC Lines	0	Disconnects	0
Multi-Term. DC	0	ZBRs	0
Areas	1	Islands	1
Zones	1	Interfaces	0
Substations	0	Injection Groups	0

Case pathname: test sys7b.PWB

Case Totals (for in-service devices only)		
	MW	Mvar
Load	3.7	2.3
Generation	3.8	2.4
Shunts	0.0	0.0
Losses	0.1	0.1

Generator Spinning Reserves	
Positive [MW]	Negative [MW]
1996.2	3.8

Negative MW Loads and Generators		
	MW	Mvar
Load	0.0	0.0
Generation	0.0	0.0

Slack Buses:
Bus 1 (1); in Area 1 (1)

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Fig 5-16 The Power Loss when put DG on bus 7 for method 2

Here, So, here, for bus 6, it is 0.13MW, for bus 7 it is 0.12MW.

5.2 Comparison of the two method:

The method 1 start with $\frac{\partial P_L}{\partial P_i}$, while the method 2 start with $\frac{\partial P_L}{\partial P_{DGi}}$. The idea for where it started are different. Comparing with the method 1, the mtheod2 are more directly on getting the result. It is because what we need to get is the P_{DG} . But, as the relationship equation $P_i = P_{DGi} - P_{Di}$. We could find the P_{Di} in the equation is a known matrix. In other words, it is a constant. So, whatever you do the derivative by P_i or by P_{DGi} , they should be the same in Mathematics aspect. In this way, we could easily find out that the Optimized result of Power loss for the two method are the same.

However, we are still able to find some differences between the two method that may involve the final result as follow:

First of all, we explain that the two method have the same meaning in Mathematics aspect. But, while we are now analysis on a physical problem. The Physics meaning for the P_i and P_{DGi} is different. the P_i is the different between all the input power and the demand as the P_{DGi} is only a part of the input power. Why we use $P_i = P_{DGi} - P_{Di}$ is only because we want to set up a convenient modeling for Mathematics iteration. This process must cause some difference while calculating.

Secondly, We could see that in the method 1. We use the directly do the partial derivative $\frac{\partial P_L}{\partial P_i}$ to get the equation:

$$P_i = \frac{1}{\alpha_{ii}} [\beta_{ii} Q_i - \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij} P_j - \beta_{ij} Q_j)]$$

In the method 2, before we do the partial derivative, we have consider the Q, the reactive power. We set up the relationship between Q and P_{DG} as following:

$$Q_i = Q_{DGi} - Q_{Di} = a P_{DGi} - Q_{Di}$$

This make the Q participate into the optimization process. As we all know that The Q, the reactive power is also a part in the power flow process, the method 1 which ignore the reactive in getting the optimal DG size and location should have bring in a little difference with the reality value for the final result.

Then, even we get the same result in the two method, the optimal location and the approximate power loss are different. We were seeking for the reason of this situation and then find out that because we use the Power World simulator to do the calculation of power flow. As the Power World Simulator are always trying to do an rounding (Half adjust) while Matlab will do an exactly calculation. And as we use the Matlab to do the approximate power loss and use the Power World to do the exactly power loss. If we see the figure 5-4,5-5, 5-12,5-13, the Matlab calculator show the result as in 0.000001. But in figure5-8, 5-9 ,5-14, 5-15, the Power World calculator show the result as in 0.1. It is easy understand why we get the difference in Approximate power loss but the same in final power loss.

What is more, when we compare the two method using the calculation by Matlab, which is the approximate power loss calculation. we could find the difference between the P_{DG6} and P_{DG7} (the 1st and 2nd optimal point) is 0.000184 for method 1 and 0.000020 for the Method 1. It is not hard to see that in the method 1 the optimal point would be easier confused than in method 2. And also, if we open the detail result sheet, we could calculate the power loss by add all the bus together. we will find the different. This is a important evidence that the method 2 is the better choice than the method 1. Because our propose is to find out the optimized DG location, we do not want to find a second optimal location by any reason.

5.3 Application:

We compare the two methods above. And now, we find out the better method is method 2. So, in this section, we will use the method 2 to analysis another test bus (13 bus) , and we also run Power World with the model, we could get the total power loss is 1 MW . The modeling is shown below:

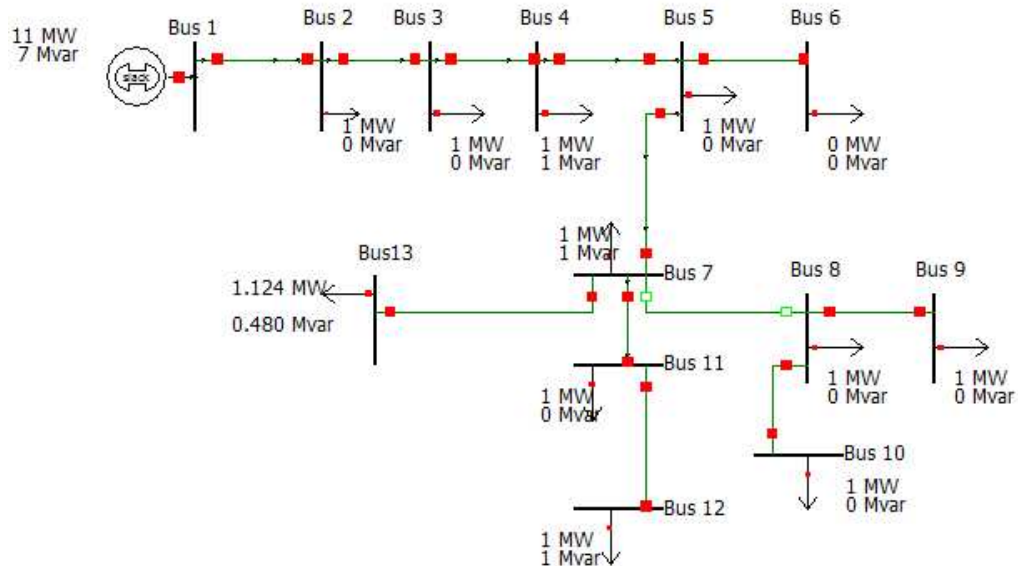


Fig 5-17 The modeling for resident area

Case Totals (for in-service devices only)		
	MW	Mvar
Load	8.2	4.7
Generation	8.6	5.0
Shunts	0.0	0.0
Losses	0.4	0.3

Fig 5-18 the original loss for resident area

Then , we get the data from the power flow result by Power World Simulator.

And we input the data into the Matlab code for method 2. In this way, we could get the result for DG size and the relationship between the power loss and P_{DG} .

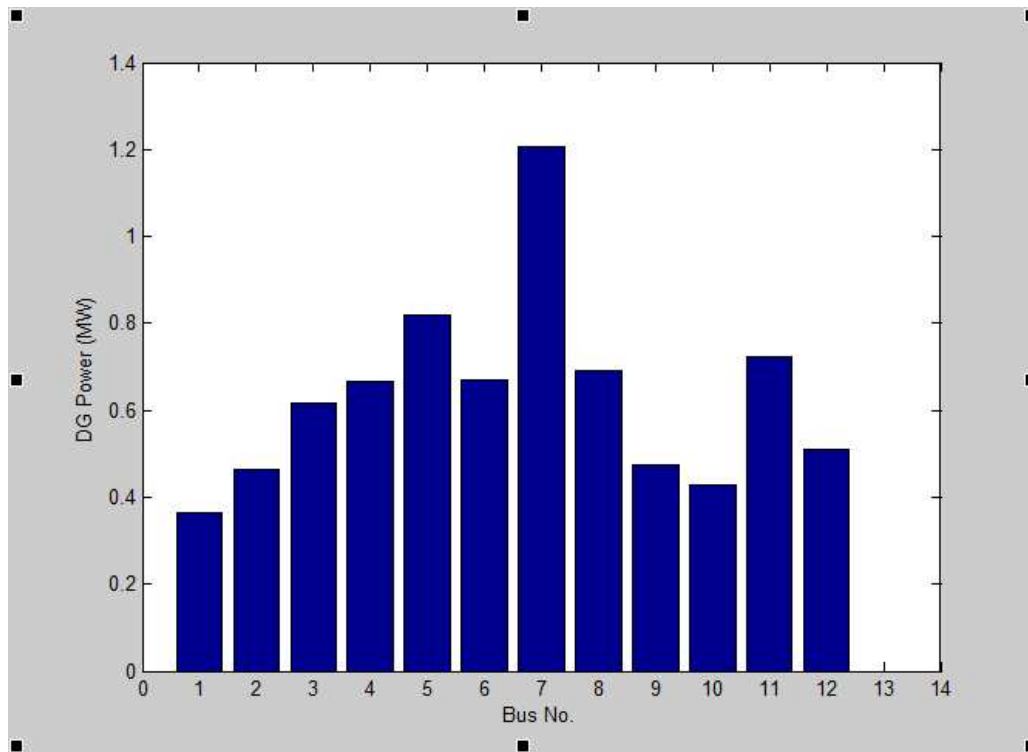


Fig 5-19 Result for resident area on relation by bus No. and P_{DG}

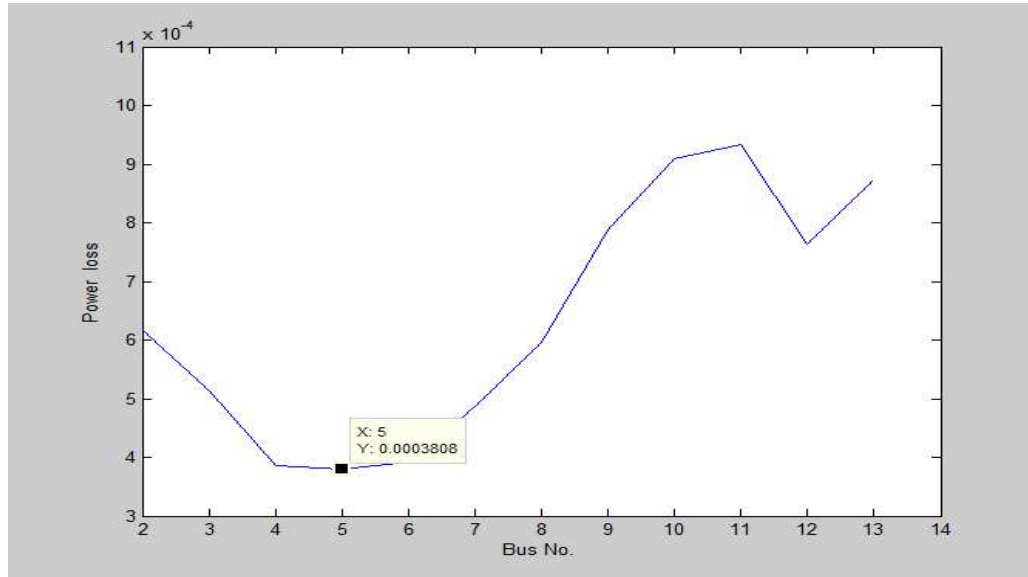


Fig 5-20 Result for resident area on relation by P_{DG} and P_L

Look at the figure5-20, we could see that the bus 5 is the optimized point for

DG location. SO we revise the figure 5-17 to add the DG as follow:

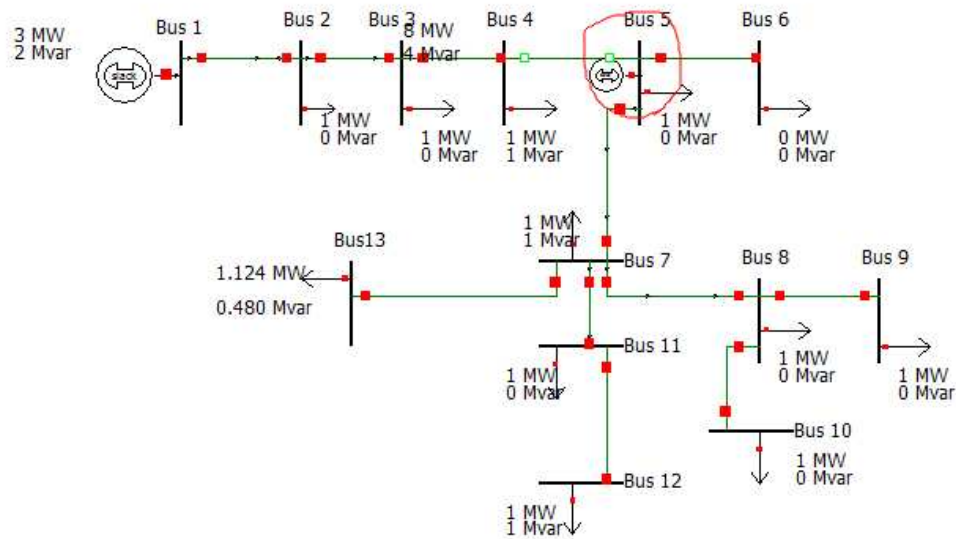


Fig 5-21 Add DG on bus 5 for resident area modeling

Then, we run the modeling and get the result and find out the following:

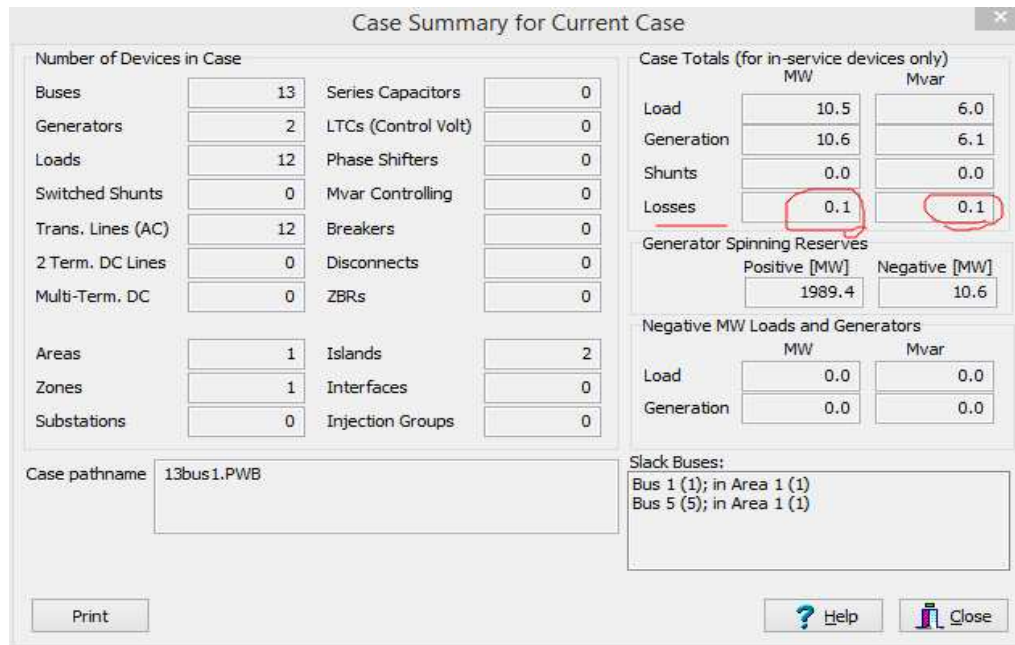


Fig 5-22 The Power Loss when put DG on bus 5 for 13 bus

From the figure, we could see that we get a perfect result on reducing the Power loss for the system.

Then, following the above case , we take out another 34 bus system (as following figure 5-23 modeling.

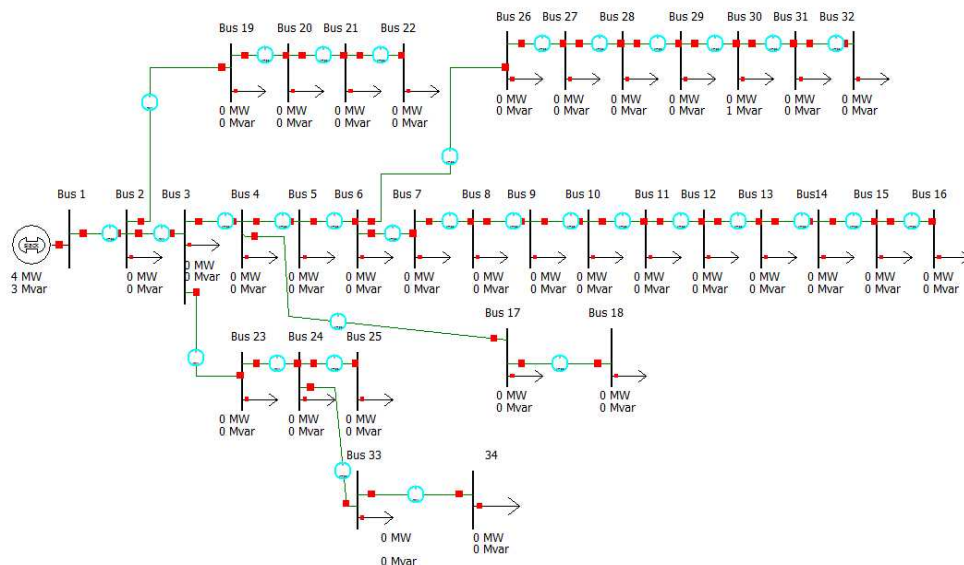


Fig 5-23 34 Bus system modeling

Also, we first get the power flow calculation and get (figure 5-24) :

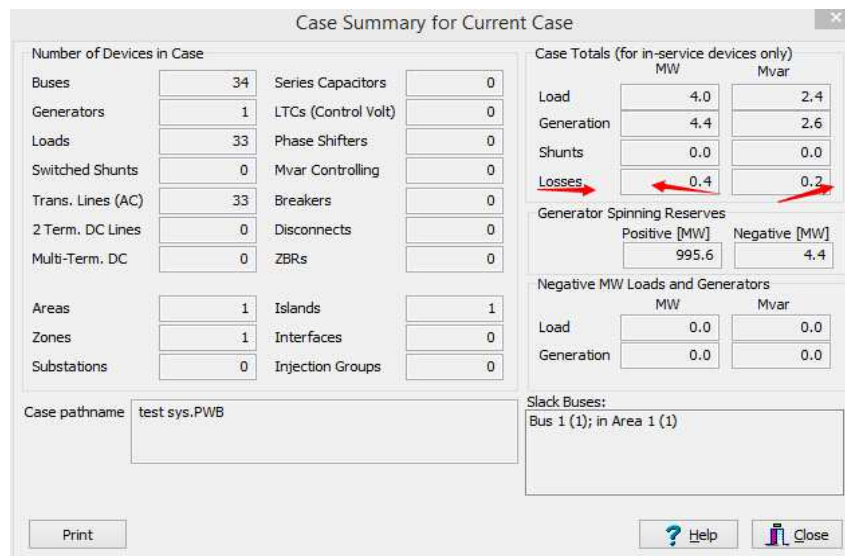


Figure 5-24 Power flow result for original 34 bus system

Then, we still use the Matlab to get optimal location for DG. And we find the Bus 17 is the optimal location and the optimal size is 3MW. So we add this generator to Bus No.17 on the system. And finally we get the power flow calculation result as following (figure 5-25):

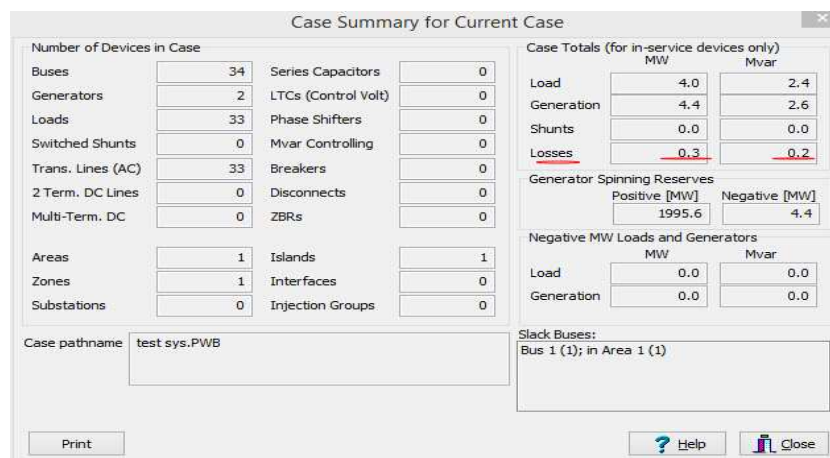


Figure 5-25 Power flow result after adding the DG on 34 bus system

From the figure 5-25, we find is the power loss of the system is being reduced from 0.4MW to 0.3MW. This is the result we need.

Chapter 6 Conclusion:

In this paper, we use two new methods to do the optimization in order to get the optimal size and location for DG in the power system. The two methods are based on the IA (Improve of Analysis) method. Then, we use the Power World and Matlab together to get the result.

We could divide the paper into two parts. The first part is analysis part. in this part we have two analysis methods. And then, the second part is simulation parts. We use two simulation tools. Then, we make a comparison both on the two methods and two tools. At last we apply the better method in another system for the optimal the DG size and location.

In comparing the two methods, we list some similar point first, and then find some differences that may influence the result (on choose the optimal location). Then, in the comparison of two method. We could find out the advantage and disadvantage of two tools.

For the Power World, we find its convenience in set up the modeling, and its friendly operation on power flow calculation. However, its inexactly calculation result will lead final result to an unclearly situation For the Matlab, we have to write

and correct the problem of the Matlab code. It will take us quite a long time. But, its result accuracy and exhibits are very good for common research.

Finally, we recognize the method 2 is a better method to get the optimal size and location for DG. Because it has a direct Mathematics thinking. Then it has a more complete consideration. At last, it has a better choice scheme of the optimal DG location. All these make the method 2 a better choose for solving the DG problems.

Chapter 7 Future works:

As we compared two methods and use the better one to solve a reality problem, we get a good result for the work. However, we still need to do further works on this topic.

First of all, when we compare the two method. We start from the old methods themselves. We step back to the theoretical induced process to prove that the base theory which support the two method is right. However, there are some mathematic approximation in inducing process. As the comparison result did not show a big difference, we could not may an exact conclusion that the second method is always the better choice to solve the DG problem. So, it is better to have a clear check of the mathematics inducing process in the future and trying to get a better mathematics theory to refine the inducing process in order to get the inducing result less relay on the approximation. Because the more approximation we made, the less accuracy we get for the result.

Secondly, we should realize that the accuracy of result shown in Power World is not very clear. Though we could get the difference in the detail result table, the Case Summary table shows the unclear result. We have to use the Matlab

result curve to see the difference. It may confuse the reader of this paper. So, we should step back to think about Power World software itself. Although, we know that the Power world is more strong and more friend in calculating the power flow and it is easy to get the result. However , we need to consider another software that are similar in the function of Power World but has a clearer result showing table. In this way, we could have a transparent result showing to support our result.

Third, we could find out that the system that we use as modeling is a little idealized. the following figure(Figure 7-1) is a part of Fig 5-1. In this figure below, we

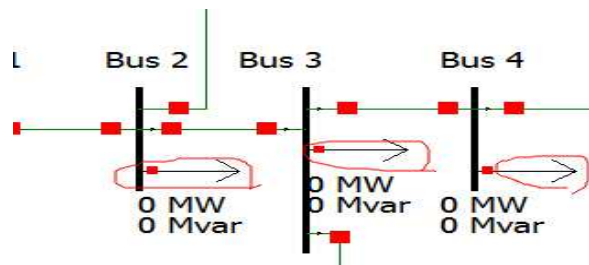


Fig 7-1 Bus details

circle out the load on each bus. There is nothing except the load on each bus.

However, there are many conductor and inductor in the reality electrical power productions and projects. In reality, many electrical parts in power system contain the conductor and inductor(like some light contain inductors and some battery include conductor). What is more? The transformer are always include in the power system. These must influence the result a lot.. This also make the result not that clear. In the future work, we need to think about solving the modeling with conductor and inductor.

In that situation, method should be changed a little. But it will bring these methods more close to the reality power electrical productions and projects. So, we will find some new modeling with conductor and inductor on buses and find the DG solution for them.

Finally, as the model we use in the paper is at small scale. We should find a bigger power system as research modeling for being closer to reality.

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